GeoStudio Example File
Simulating Consolidation with SEEP/W

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Introduction

Consolidation testing is conducted by many engineers to determine stress-strain relationships. Consolidation occurs when a soil is subject to loading, leading to changes in the volume of the soil-pores over a period of time following the initial loading of the soil (Balasubramaniam and Brenner, 1981). The objective of this example is to simulate the conventional consolidation process using SEEP/W.

Background

The governing differential equation for flow through an elemental volume in two-dimensions as described in the SEEP/W documentation is:

$$\frac{\partial}{\partial x}\left(k_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial h}{\partial y}\right) = \frac{\partial \theta}{\partial t}$$ Equation 1

where $k_x$and k_y are the hydraulic conductivity in the x- and y-directions, respectively, h is the total head, θ is the volumetric water content, and t is time.

For the one-dimensional consolidation discussion here, we will deal only with the flow in the vertical direction (y). The flow equation then becomes:

$$\frac{\partial}{\partial y}\left(k_y \frac{\partial h}{\partial y}\right) = \frac{\partial \theta}{\partial t}$$ Equation 2

In a consolidation analysis, equations are generally solved using pore-water pressure instead of total head. The relationship between total head (h) and pore-water pressure (u) is:

$$h = \frac{u}{\gamma_w} + y$$ Equation 3

where y is the elevation and γ_w is the unit weight of water. Substituting for h in Equation 2 then gives:

$$\frac{\partial}{\partial y} + \left(\frac{k_y \partial u}{\gamma_w \partial y} + k_y \frac{\partial y}{\partial y}\right) = \frac{\partial \theta}{\partial t}$$ Equation 4

The elevation is a constant and, therefore, the derivative of y with respect to y is unity. Then:

$$\frac{\partial}{\partial y} + \left(\frac{k_y \partial u}{\gamma_w \partial y} + k_y\right) = \frac{\partial \theta}{\partial t}$$ Equation 5

$$\frac{\partial}{\partial y} + \left(k_y\left(\frac{1}{\gamma_w}\frac{\partial u}{\partial y} + 1\right)\right) = \frac{\partial \theta}{\partial t}$$ Equation 6

The first part in the inner bracket represents the changes in volumetric water content arising from variations in pore-water pressure while the second part represents changes in volumetric water content due to gravity. In consolidation analyses, the pore-pressure variations are often due to some external loading, while the gravitational term represents the drainage in the soil due to variations in elevation.

In order to have only one primary unknown, it is necessary to write the volumetric water content in terms of pore-pressure. This can be accomplished using:

$$\partial\theta = m_w \partial u$$ **Equation 7**

where m_w is the slope of a volumetric water content function (or soil water characteristic curve). When the pore-water pressure is positive, m_w is equivalent to m_v, the coefficient of volume compressibility. Stated another way, m_v is defined as a volumetric change (ΔV) resulting from a unit change in pressure (Δp):

$$m_v = \frac{\Delta V}{\Delta p}$$ **Equation 8**

Under saturated conditions, when it is assumed that water is incompressible, then $\Delta\theta$ is equal to ΔV, changing Equation 7 to:

$$\partial\theta = m_v \partial u$$ **Equation 9**

Now, substituting for $\partial\theta$ in the flow equation:

$$\frac{\partial}{\partial y}\left(k_y\left(\frac{1\partial u}{\gamma \partial y} + 1\right)\right) = m_v \frac{\partial u}{\partial t}$$ **Equation 10**

By moving m_v to the left side, the equation becomes:

$$\frac{\partial}{\partial y}\left(\frac{k_y}{m_v}\left(\frac{1\partial u}{\gamma \partial y} + 1\right)\right) = \frac{\partial u}{\partial t}$$ **Equation 11**

If we ignore the gravitational term and assume that the material properties are constants, then the equation becomes:

$$\frac{\partial}{\partial y}\left(\frac{k_y}{m_v}\left(\frac{1\partial u}{\gamma \partial y}\right)\right) = \frac{\partial u}{\partial t}$$ **Equation 12**

$$\frac{k_y}{\gamma m_v} \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t}$$ **Equation 13**

The front expression is the same as the coefficient of consolidation (C_v):

$$C_v = \frac{k}{\gamma_w m_v}$$ **Equation 14**

Substituting C_v into the equation forms the classic form of the Terzaghi consolidation equation (Terzaghi, 1943):

$$C_v \frac{\partial^2 u}{\partial y^2} = \frac{\partial u}{\partial t}$$ Equation 15

The implication here is that the SEEP/W transient flow formulation intrinsically is equivalent to the Terzaghi consolidation formulation under certain conditions. The practical implication is that SEEP/W can be used to analyze the dissipation of excess pore-water pressures in a consolidation process.

SEEP/W always includes the gravitational effect, but, in a thin lab sample, this effect is very small relative to the incremental pore-water pressure arising from an applied load, so it is still possible to make comparison with closed-form consolidation solutions.

Numerical Simulation

This example simulates the consolidation of a 0.05 m thick lab sample using a one-dimensional analysis (Figure 1). The soil is saturated at all times, so a Saturated-Only material model is used. The saturated hydraulic conductivity is set to 1x10-8 m/sec, with an m_v of 0.005 1/kPa.

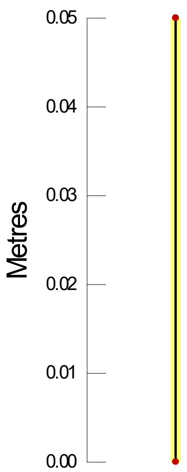

Figure 1. Model domain.

The hydraulic conductivity, or coefficient of permeability, is related to C_v and m_v by Equation 14. The coefficient of consolidation (C_v) then is:

$$C_v = \frac{k}{\gamma_w m_v} = \frac{1 \times 10^{-8}\ m/sec}{(9.81\ kN/m^3)(0.005\ 1/kPa)} = 2.04 \times 10^{-7}\ m^2/sec$$ Equation 16

The time factor (*T*) to reach 50% consolidation is 0.197. The time (*t*) then to reach 50% consolidation in the lab sample here is:

$$t = \frac{TH^2}{C_v} = \frac{0.197(0.025)^2}{2.04 \times 10^{-7}} = 604\ sec$$ Equation 16

For 25% consolidation, *T* is 0.49 making *t* = 150 sec and for 75% consolidation, *T* is 0.477 making *t* = 1460 sec.

These values are specified in the time sequencing to ensure these results are available in the Results window (Figure 2).

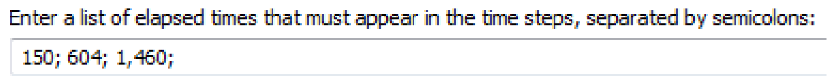

Figure 2. List of desired time steps.

The initial excess pore-water pressure is activated at 10 kPa; this is similar to applying a load of 10 kPa. Zero pressure boundary conditions were also applied to the top and bottom nodes to ensure that the soil remains saturated. The total simulation time was set to 3 hours, with 10 time steps increasing exponentially with an initial time step of 2 seconds.

Results and Discussion

The dissipation of the excess pore-water pressures follows the typical picture from a conventional consolidation test as shown in Figure 3. Figure 4 shows the isochrones for *t* equal to 150, 604 and 1460 sec. This graph data was pasted into EXCEL and the areas under the curves were computed by simply adding up all the values, as the element spacing is a constant 0.001 m. The area under the curve was then subtracted from the total area to calculate the degree of consolidation.

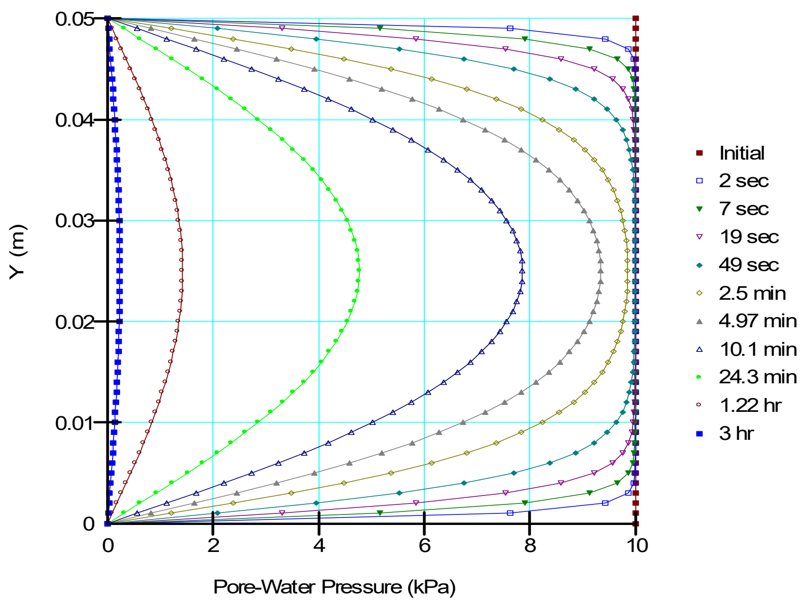

Figure 3. The dissipation of the 10 kPa excess pore-water pressure.

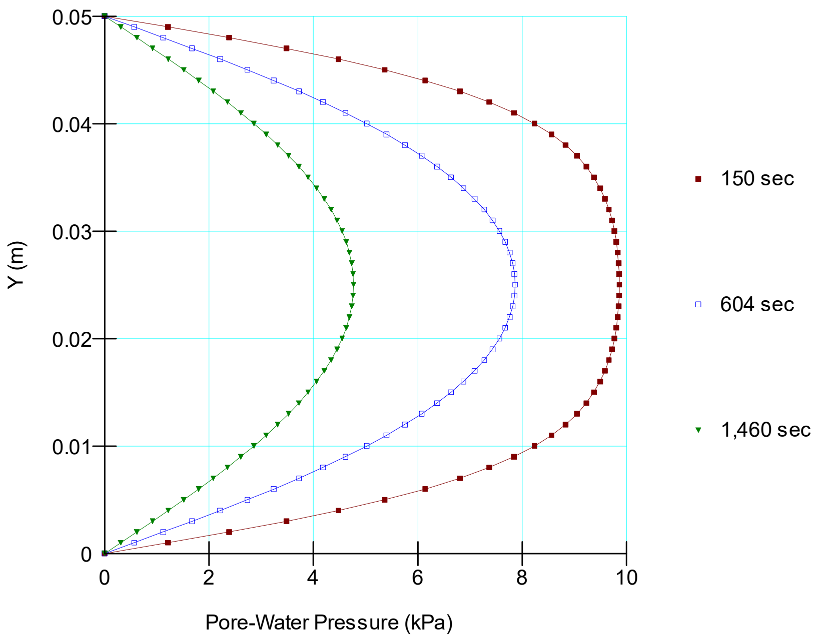

Figure 4. The isochron representing approximately 25%, 50% and 75% consolidation.

The resulting degree of consolidation calculated using the simulated curves are compared with the closed-form solutions in Table 1. The degrees of consolidation from SEEP/W are similar to the published closed-form solutions, especially at the lower degrees of consolidation.

Table 1. Comparison of closed-form and SEEP/W results for degree of consolidation.

T (time factor)	*t (sec)*	*Percent consolidation Closed-form*	*Percent consolidation SEEP/W*
0.049	150	25	24
0.197	604	50	48
0..477	1460	75	69

It is well known that the closed-form Taylor-Series solutions to the consolidation equation are fairly accurate up to about 50% consolidation. Beyond 50% consolidation the accuracy of the closed-form solution is reasonable but not as good as for lower degrees of consolidation. This shows up in Table 1. The difference in the comparison increases as the degree of consolidation increases. Based on consideration of the two formulations and resulting solutions, the SEEP/W results are more formal and, consequentially, considered more accurate.

The close similarity in form and magnitude between the SEEP/W results and the closed-form solutions lends credence to the fact that SEEP/W has been correctly formulated.

Summary and Conclusions

This illustration demonstrates that SEEP/W can be used to model the dissipation of excess pore-water pressures associated with consolidation. The solution is tightly tied to the specified m_v associated with the volumetric water content function and the specified hydraulic conductivity

References

Balasubramaniam, A.S. and Brenner, R.P. 1981. Chapter 7 – Consolidation and settlement of soft clay. *In* Soft Clay Engineering. Developments in Geotechnical Engineering, 20: 479-566.

Terzaghi, K. 1943. Theoretical Soil Mechanics. Wiley, New York.