GeoStudio Example File

Verification – Beams and Bars in a Frame

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Introduction

The purpose of this example is to verify the behavior of beams and bars in SIGMA/W when the structural elements are not connected to any other regular soil elements. All of the results are verified by comparison with hand calculations.

Numerical Simulation

The model domain comprises seven geometry lines that are used to represent various scenarios of structural beams (Figure 1). There are a total of seven load-deformation analyses in the Analysis Tree (Figure 2), all of which solve a single load step.

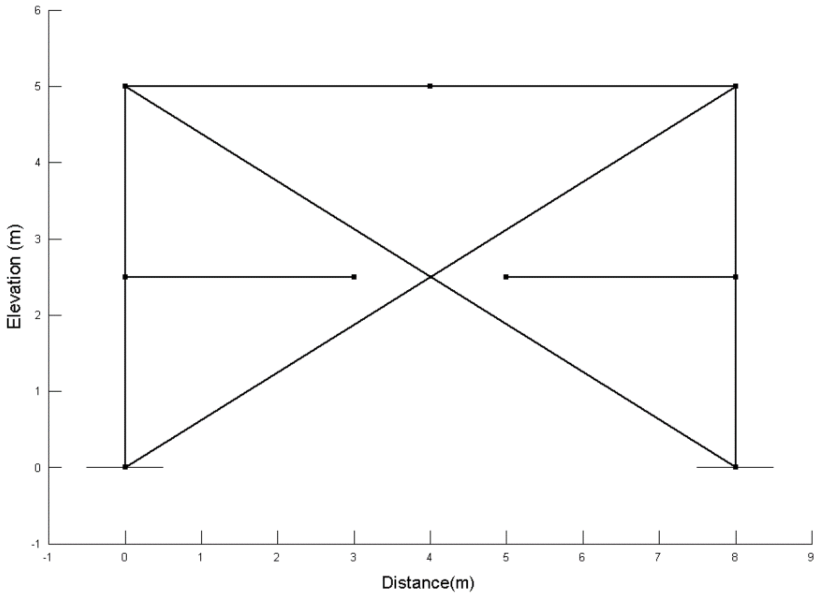

Figure 1. Problem configuration for the analyses.

The first analysis consists of a simple beam with a point load (Figure 3). The beam is 8 m in length and is fixed in the x and y directions at the left end using a displacement-type boundary condition with specified values of zero. At the right end, the beam is on a roller that is simulated by setting the y-displacement to zero and leaving the x-displacement undefined. The point load is specified as a Y-Force boundary condition equal to -1000 kN. The minus sign refers to the negative y-direction.

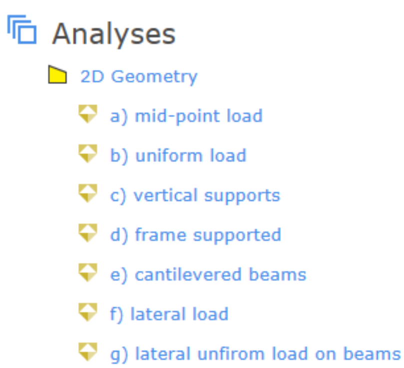

Figure 2. Analysis Tree for the Project.

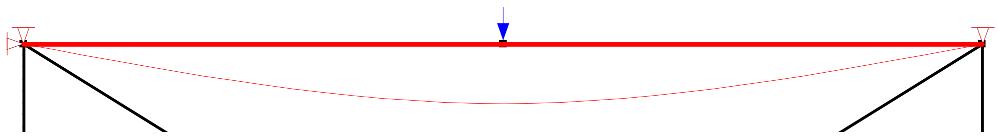

Figure 3. Simple beam with a point load (Analysis a).

The second analysis (analysis (b)) consists of a simple beam with a uniform load (Figure 4). The beam is fixed again in the x and y directions at the left end, and is 8 m in length. At the right end, the beam is on a roller that is simulated by setting the y-displacement to zero and leaving the x-displacement undefined. The uniform load is specified as an x-y boundary stress with Y-Stress equal to -100 kPa. The minus sign refers to the negative y-direction.

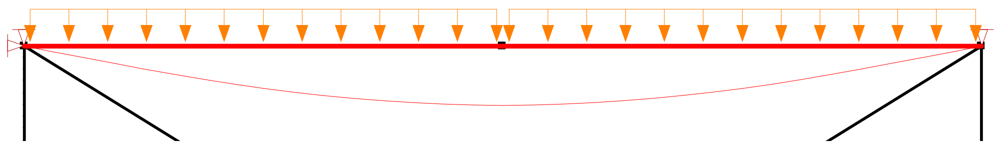

Figure 4. Beam with uniform load (Analysis b).

In Analysis c, structural bars are substituted as legs for the beam in Analysis 2 (Figure 5). The purpose is to test the functionality of the bars in this situation. The legs at each end consist of two bars. To keep the structure stable, zero-displacement boundary conditions are required at all three points on the left and at the mid- and lower-points on the right. The upper right point is stabilized by the beam being connected to the left zero-displacement boundary condition.

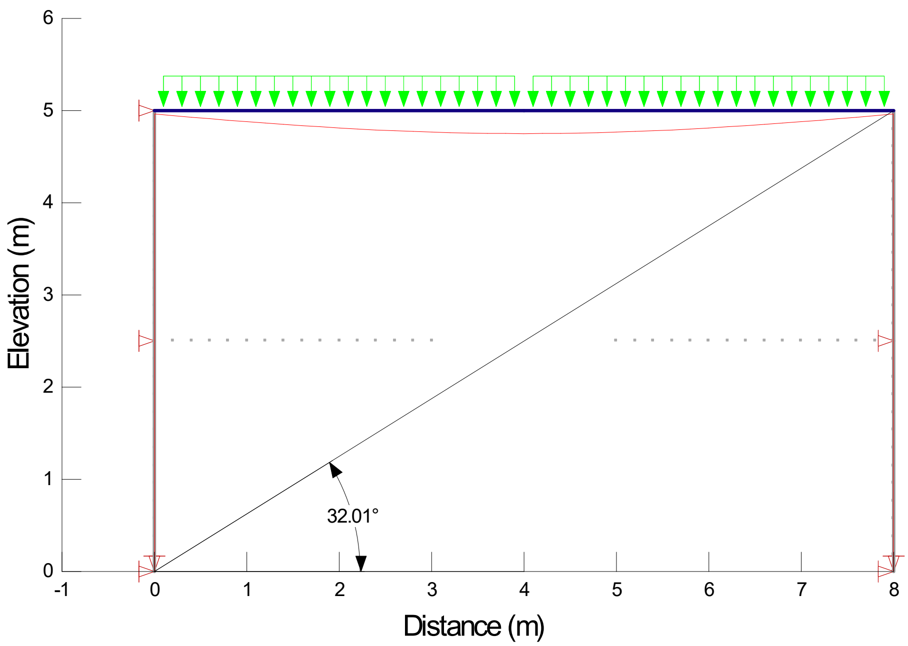

Figure 5. Beam on legs (Analysis c).

In Analysis d, the beam is placed on a frame where the cross-braces are modeled as bars (Figure 6). Note that displacement boundary conditions are no longer required at the ends of the beam to keep the structure stable.

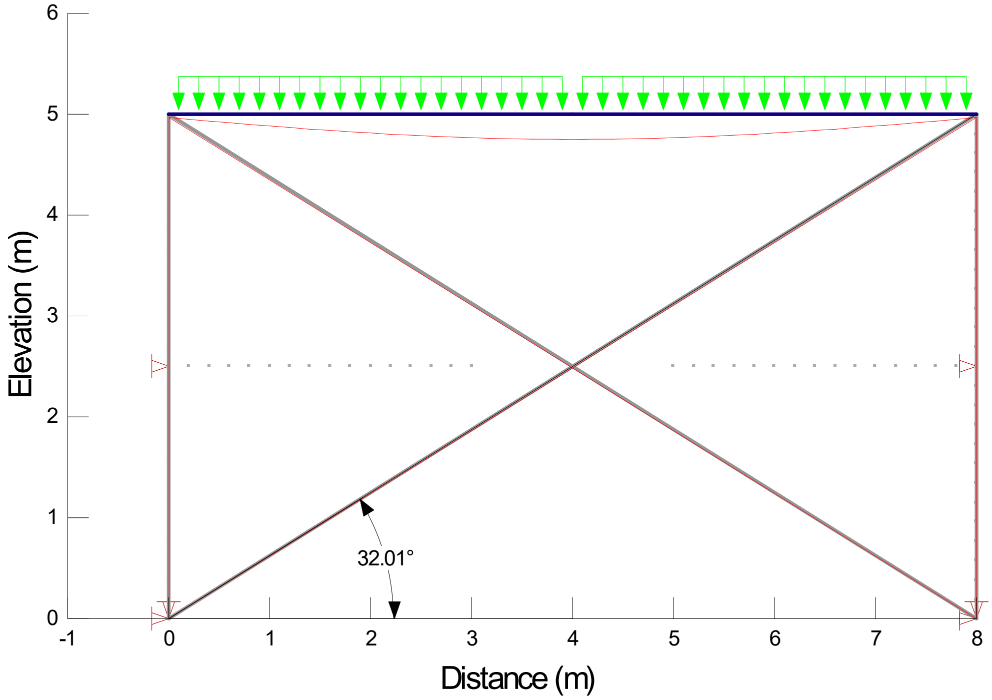

Figure 6. Beam on frame (Analysis d).

Figure 7 illustrates the use of rotation-type boundary conditions in Analysis e. Specifying the rotation as zero at the end of the extending beams makes it possible to analyze a cantilever beam. The uniform load on each cantilevered beam is 100 kN/m and the length is 3 m.

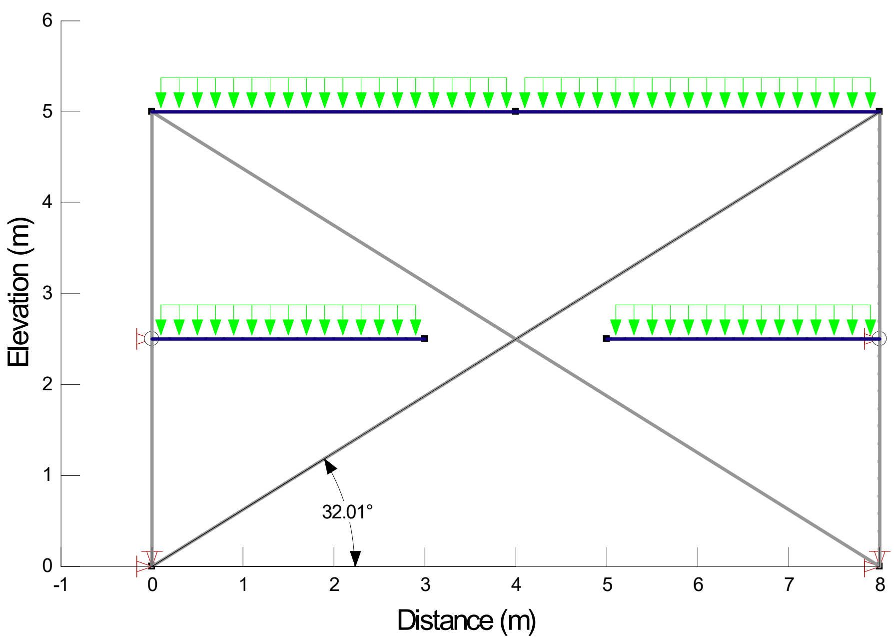

Figure 7. Case with cantilever beams (Analysis e).

In Figure 9, a lateral (horizontal) load is applied at the top left corner in Analysis f. The force is equal to 1000 kN and is applied as an X-Force boundary condition. Part of the force will go into the axial force in the top beam and the other part will go into the cross-brace. The reaction in the top beam at the right end will be resisted by the other cross-brace.

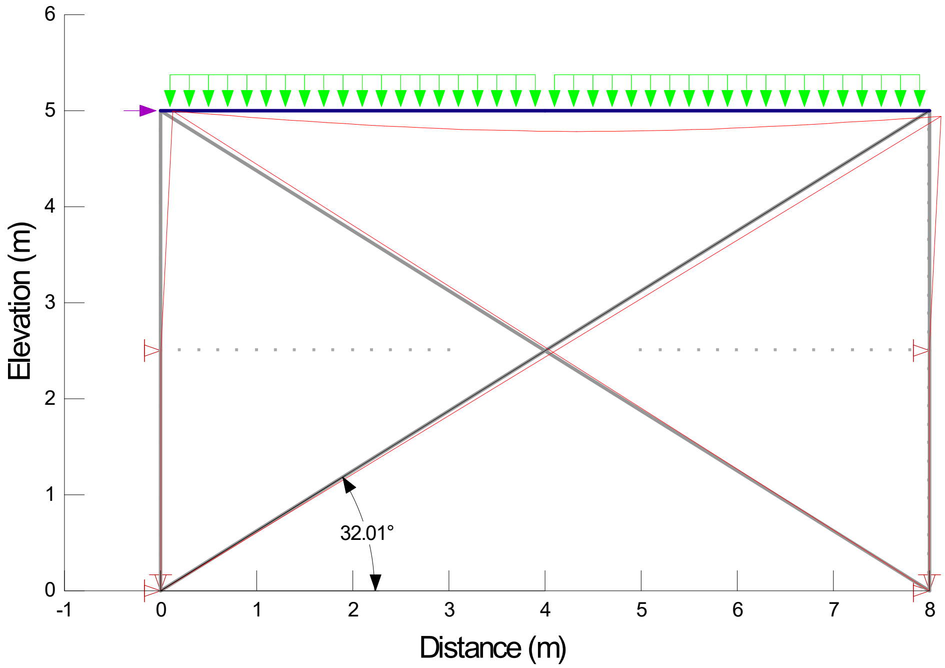

Figure 9. Frame with lateral load (Analysis f).

In the final analysis, the left vertical leg is replaced with a beam and a lateral load is applied to the vertical beam (Figure 10). The main purpose here is to check the moment distribution in two beams connected at the top corner.

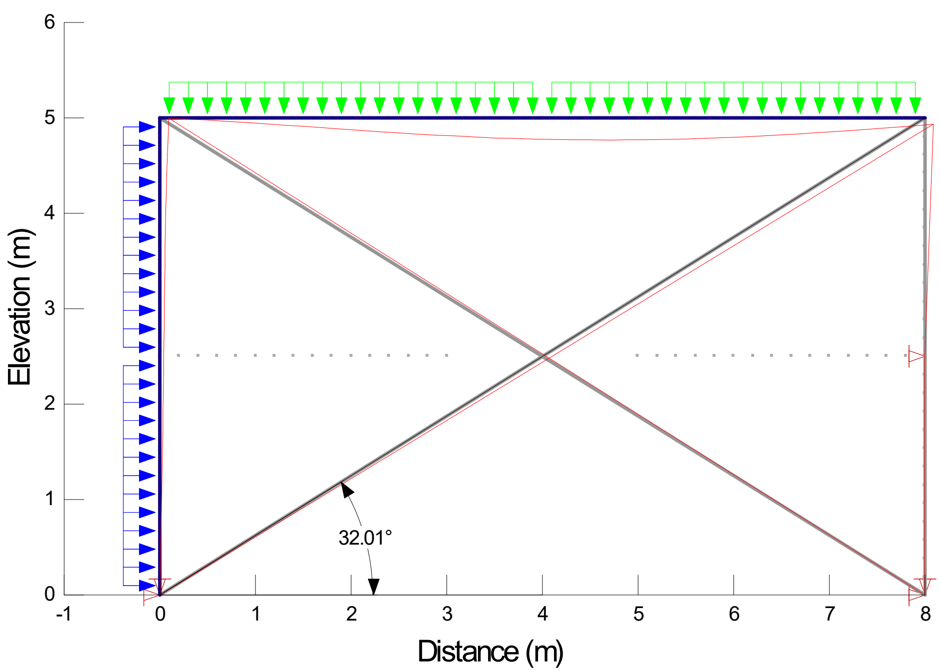

Figure 10. Frame with two beams (Analysis g).

Results and Discussion

The result of Analysis a, with a single load point at the centre of a beam, can first be calculated manually. The moment is zero at the supports and the maximum occurs at the middle of the beam. The maximum must be equal to:

$$W * \frac{L}{4} = 100 * \frac{8}{4} = 2000\ kN - m \qquad \text{Equation 1}$$

which was computed in Analysis a (Figure 11). The maximum value presented in the graph is just under 2000 kN-m due to interpolation of gauss data to nodes.

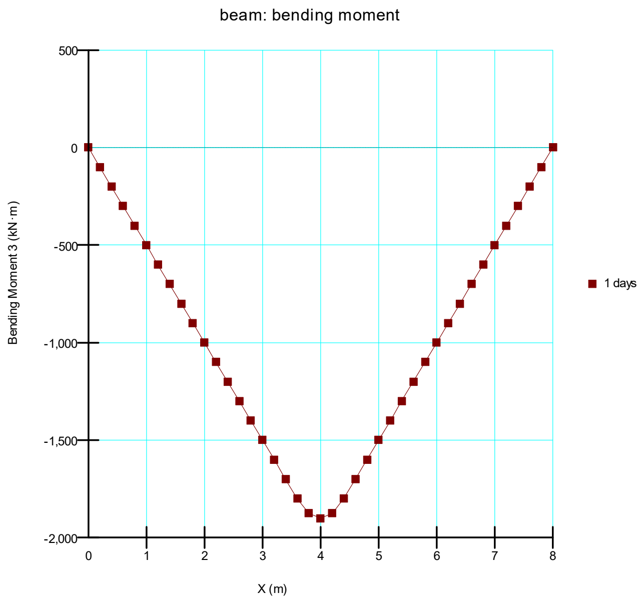

Figure 11. Moment distribution for beam with point load (Analysis a).

The maximum shear is at the ends and must be equal to:

$$\frac{P}{2} = \frac{1000}{2} = 500\ kN \qquad \text{Equation 2}$$

Figure 12 shows the correct shear distribution. The end shear values must also be equal to the support reactions. The SIGMA/W computed Y-Boundary value is at 500 kN.

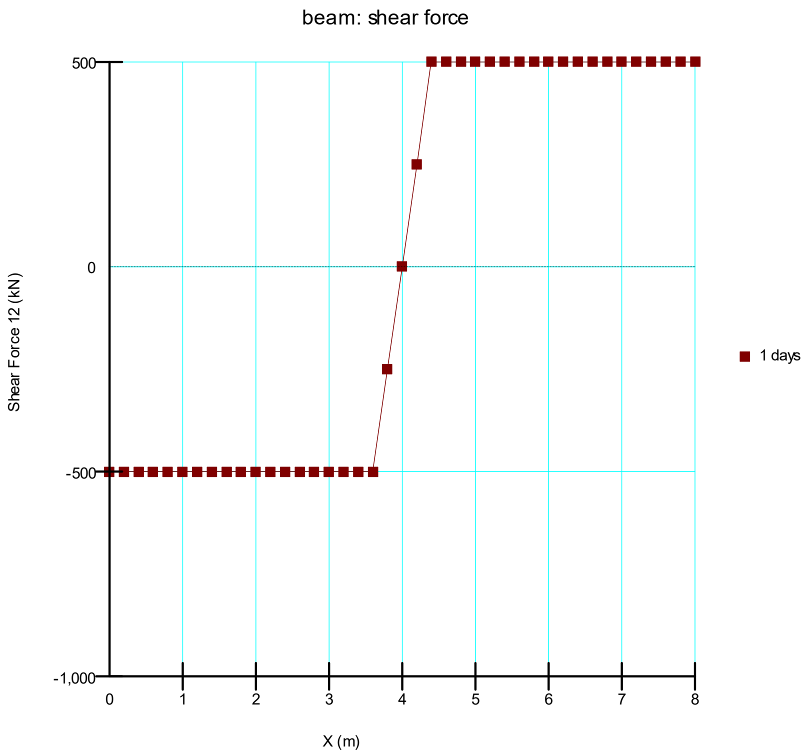

Figure 12. Shear distribution for beam with point load (Analysis a).

The maximum deflection must be at the mid-point and can be calculated as:

$$P * \frac{L^3}{(48 * E * I)} = 1000 * \frac{8^3}{\left(48 * 10^8 * 0.01\right)} = 0.01066\, m \quad \text{Equation 3}$$

The SIGMA/W computed value is 0.0107 m and occurs at the mid-point as shown in Figure 4.

For the second analysis (single beam with uniform load), the moment is zero at the supports, and the maximum occurs at the middle of the beam and must be equal to:

$$W * \frac{L^2}{8} = 100 * \frac{8^2}{8} = 800\, kN - m \quad \text{Equation 4}$$

Figure 13 confirms this to be the case.

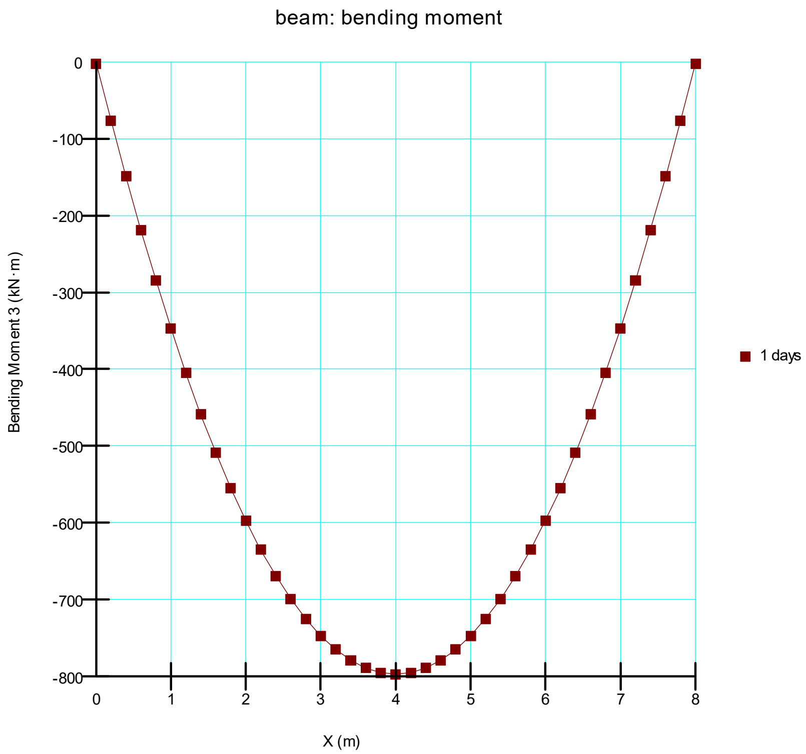

Figure 13. Moment distribution for beam with uniform load (Analysis b).

The maximum shear is at the ends of the beam and must be equal to:

$$W * \frac{L}{2} = 100 * \frac{8}{2} = 400\ kN - m$$

Equation 5

Figure 14 shows the correct shear distribution. The maximum deflection must be at the mid-point and must be:

$$\frac{(5 * w * L^4)}{(384 * E * I)} = \frac{5 * 100 * 8^4}{384 * 10^8 * 0.01} = 0.005333\ m$$

Equation 6

The SIGMA/W computed value is 0.005347 m at the mid-point as shown in Figure 4. The vertical reactions at the ends equal the summation of forces apportioned to all other nodes, verifying equilibrium of the structure.

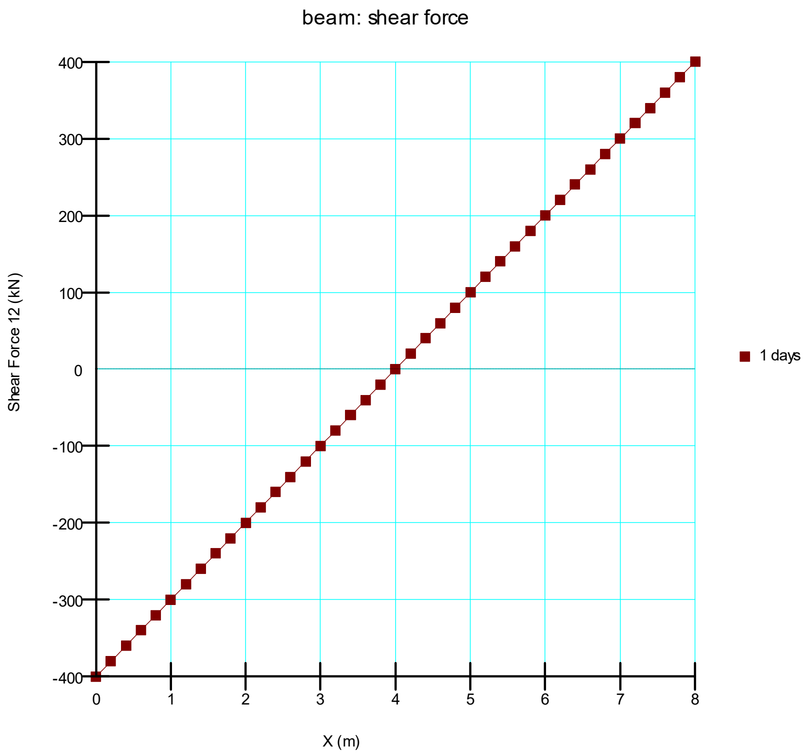

Figure 14. Shear distribution for beam with uniform load (Analysis b).

For Analysis c (beam with uniform load on legs), the SIGMA/W computed axial force in each bar must be 400 kN (half of the uniform load on the beam). The cross-sectional area is 0.01 m and, therefore, the axial stress is 40,000 kPa. The compression at the top of the leg is 0.001 m (1 mm). Therefore, the strain is:

$$\frac{0.001}{5} = 0.0002 \; or \; 0.02\% \qquad \text{Equation 7}$$

When the cross-braces are added in Analysis d, a portion of the load will now be in the vertical legs and a portion will be in the cross braces. The length of the cross braces is equal to $(5^2 * 8^2)^{1/2}$ = 9.434 m. The SIGMA/W computed forces in the leg and cross brace are 348.44 kN and 97.28 kN, respectively. The sum of 348.44 and 97.28 * 5 / 9.434 equals 400 kN, which correctly corresponds to half the uniform load on the beam.

For Analysis e (cantilever beams), the maximum moment can be calculated as:

$$W * \frac{L^2}{2} = 450 \; kN \qquad \text{Equation 8}$$

which matches the SIGMA/W graph in Figure 15. The maximum shear must be 100 * 3 = 300 kN. The curve in Figure 16 is equivalent. The maximum deflection at the free end must be:

$$W * \frac{L^4}{(8 * E * I)} = 100 * \frac{3^4}{8 * 10^8 * 0.01} = 0.0010125\, m \quad \text{Equation 9}$$

For the cantilever on the left, the SIGMA/W computed vertical displacement on the left end is 0.0007864 m and on the right end is 0.001809 m – the difference is 0.001023 m (or 1.0131 mm), which is equal to the hand-calculated value above.

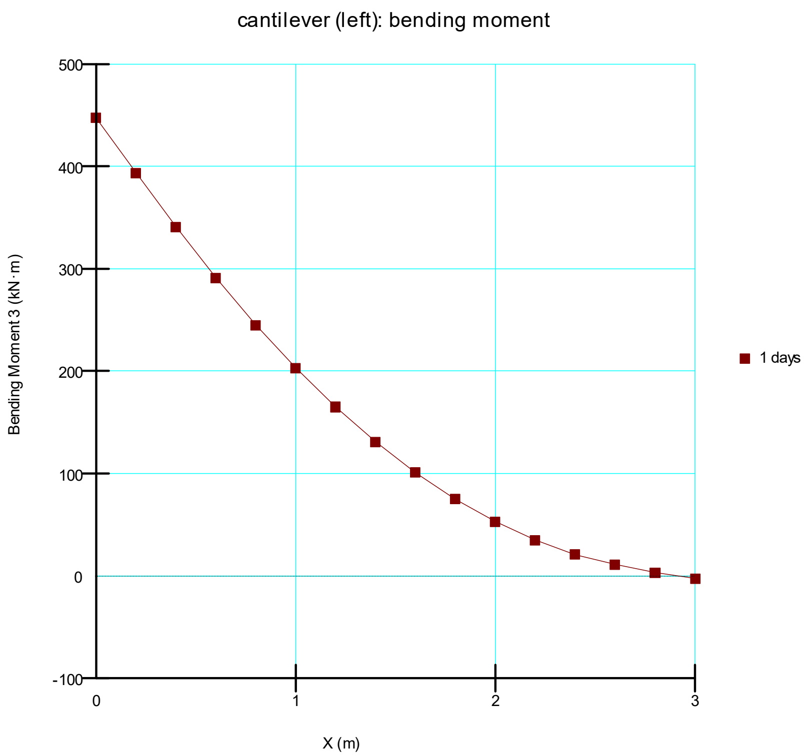

Figure 15. Moment distribution for cantilever beam (Analysis e).

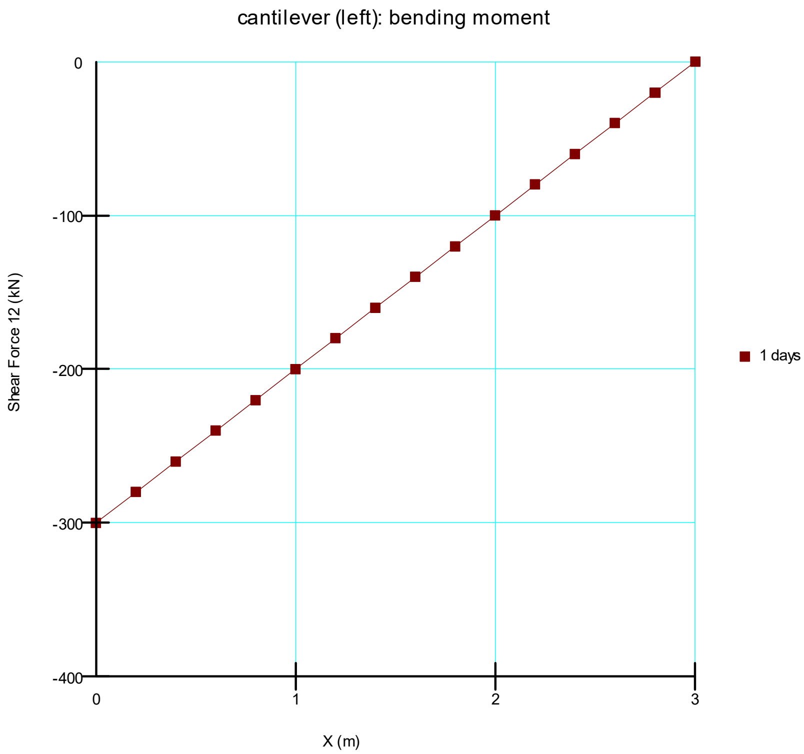

Figure 16. Shear distribution for cantilever beam (Analysis e).

The axial force transferred into the beam for Analysis f (with a lateral load) is governed by the cross bracing. The beam picks up 415 kN in compression. This is perfectly balanced by the x-component of the cross brace that is pulled into tension (-415 kN). The remaining 585 kN of x-force applied to the beam is transferred into the other cross brace.

Closed-form solutions are not readily available for Analysis 6, but the distribution is intuitively correct. In the earlier analysis, the maximum moment in the top beam was 800 kN-m. Now, the maximum moment is about 536 kN-m, due to the countervailing moment in the vertical beam. Also, the moment at the top left corner must be of the opposite sign from the moment in the beams when not connected.

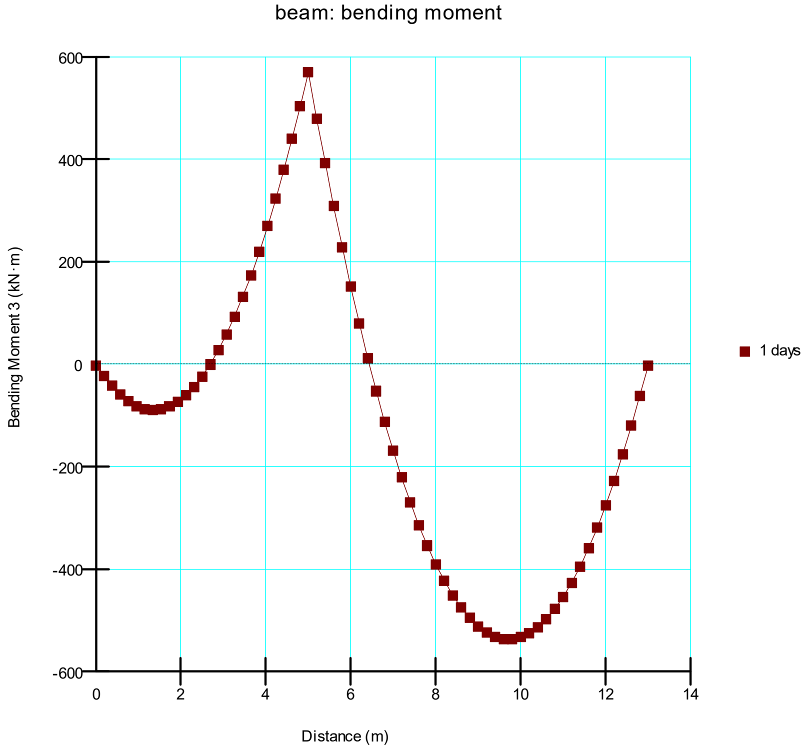

Figure 17. Moment distribution in two connected beams (Analysis g).

Summary and Conclusions

These examples and analyses confirm that the structural elements in SIGMA/W function correctly.