GeoStudio Example File
Kirsch Analytical Solution Comparison

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Introduction

Stresses and displacements are determined numerically for the case of a circular cross section of a long excavation in a linearly elastic medium subject to a biaxial stress defined by $p_{yy} = p$ and $p_{xx} = Kp$. The solutions for the stress and displacements around the circular opening are originally due to Kirsch (1898) and are summarized in Brady and Brown (2006). The problem tests the linear elastic model with applied field stresses in plane-strain conditions imposed in Sigma/W.

The elastic material is assigned the following properties:

Young's modulus (E) 1 kPa

Poisson's ratio (ν) 0.25

The in-situ stress has a magnitude of $p = 1$ kPa with $K = 0$. The radius of the hole is, $a = 1$ m.

The closed-form solution for stresses displacements around the circular opening are shown in Figure 1 and the equations below. u_r, u_θ are displacements induced by the excavation, σ_{rr}, $\sigma_{\theta\theta}$, $\sigma_{r\theta}$ are total stresses generated after the opening is excavated, and r is the radial distance from the centre of the circular opening.

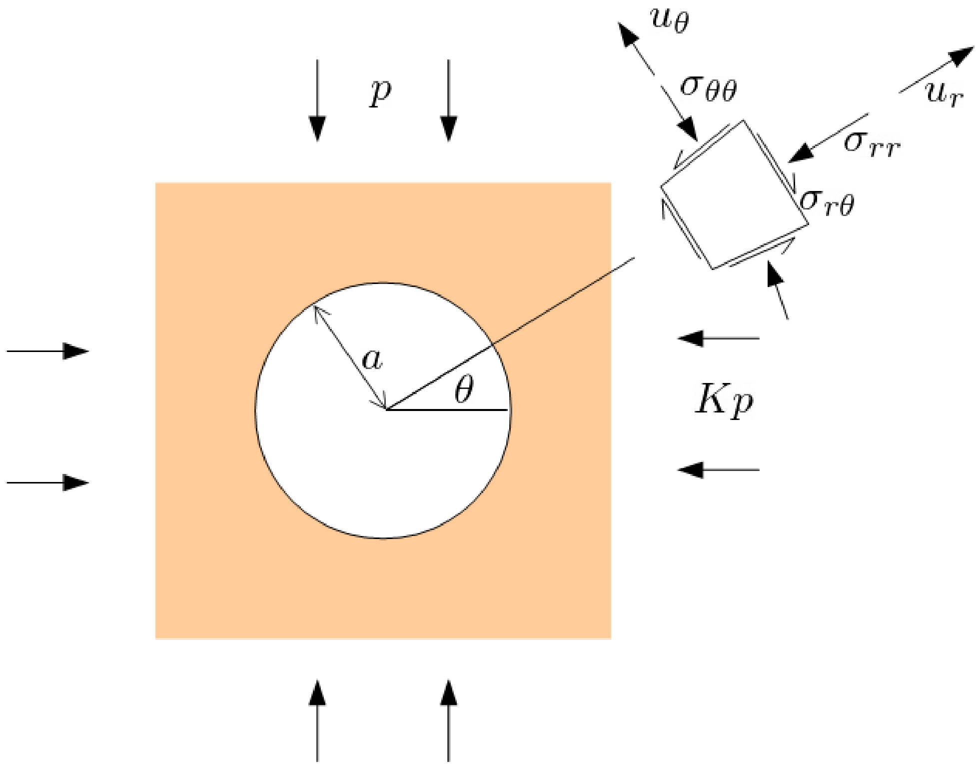

Figure 1. Problem geometry and nomenclature.

$$\sigma_{rr} = \frac{p}{2}\left[(1+K)\left(1-\frac{a^2}{r^2}\right) - (1-K)\left(1-4\frac{a^2}{r^2}+3\frac{a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_{\theta\theta} = \frac{p}{2}\left[(1+K)\left(1-\frac{a^2}{r^2}\right) + (1-K)\left(1+3\frac{a^4}{r^4}\right)\cos 2\theta\right]$$

$$\sigma_{r\theta} = \frac{p}{2}\left[(1-K)\left(1+2\frac{a^2}{r^2}+3\frac{a^4}{r^4}\right)\sin 2\theta\right]$$

$$u_r = -\frac{pa^2}{4Gr}\left[(1+K)-(1-K)\left(4(1-\nu)-\frac{a^2}{r^2}\right)\cos 2\theta\right]$$

$$u_\theta = -\frac{pa^2}{4Gr}\left[(1-K)\left(2(1-2\nu)+\frac{a^2}{r^2}\right)\sin 2\theta\right]$$

Numerical Simulation

For modelling purposes, the problem is defined by the domain sketched in Figure 2. The model takes advantage of quarter symmetry. In the analytical solution, infinite boundaries are assumed. In order to reduce the effects of the external boundary on model results, the Sigma/W model boundaries are placed at 10x the radius from the hole centre.

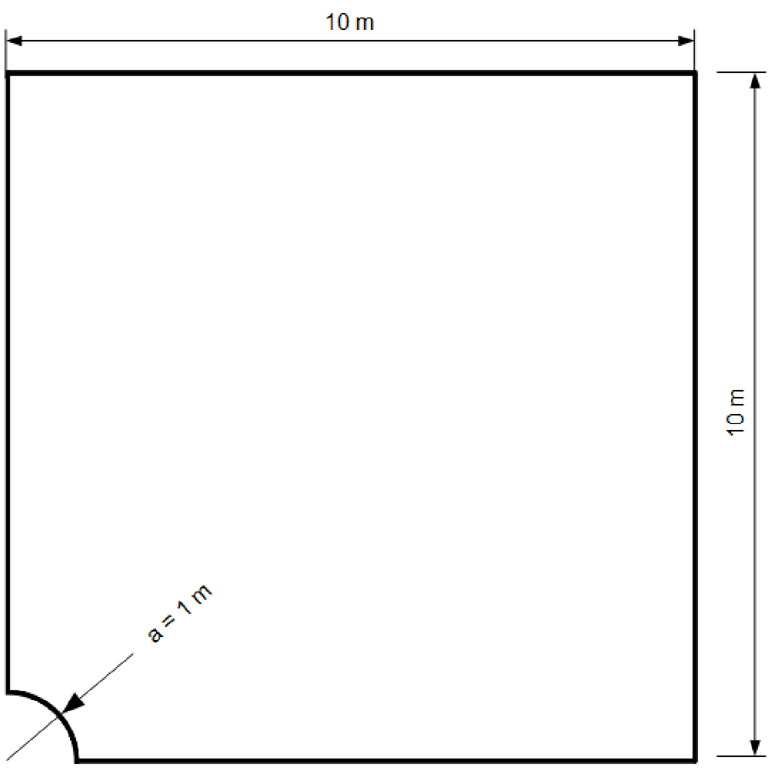

Figure 2. Model geometry.

The GeoStudio configuration for this model is shown in Figure 3. The 3D geometry tools were used to construct the geometry and a vertical section was cut through the 3D model for the 2D plane strain geometry. An artificial radius=3 m circle was introduced into the model to create more finely discretized area of the model near to the excavation.

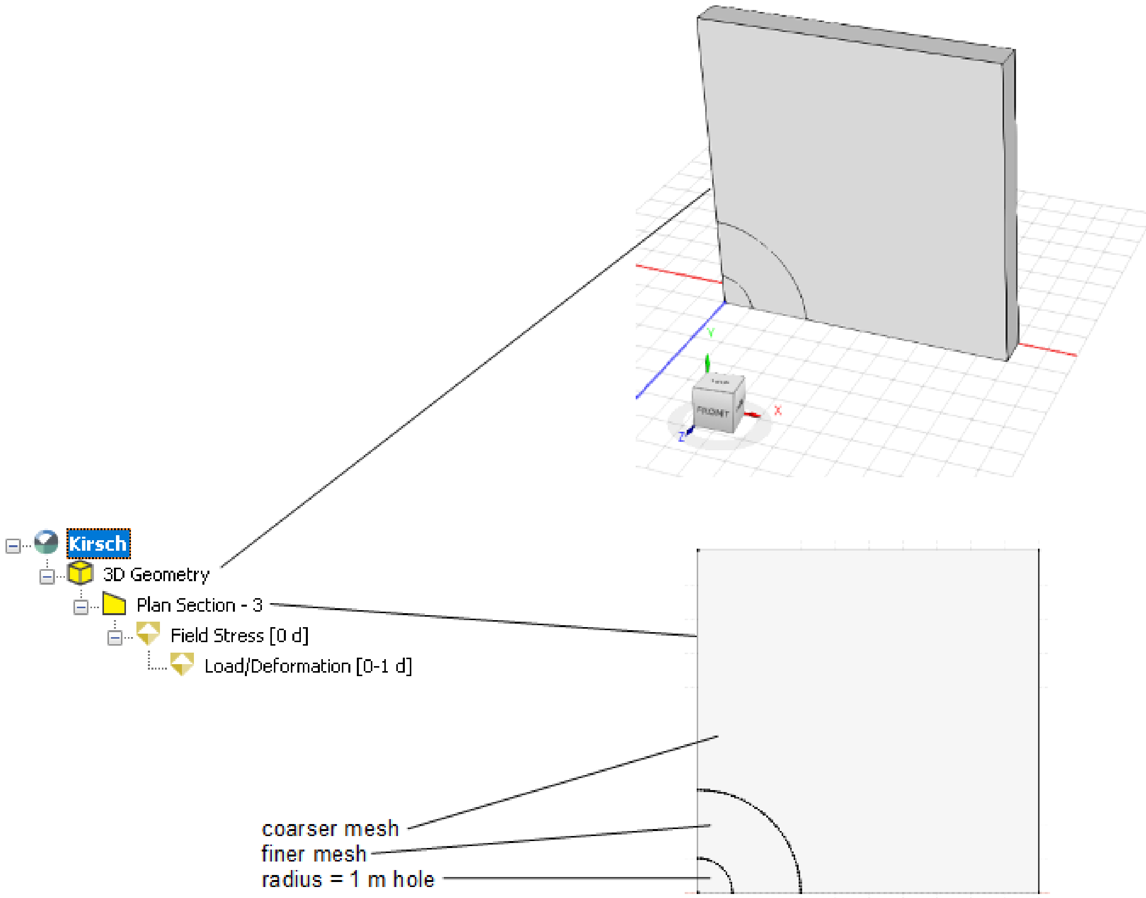

Figure 3. GeoStudio analysis tree, 3D geometry, and vertical cross section through the 3D geometry.

Figure 4 shows the In Situ analysis of the GeoStudio model. In the Define Project options, an In Situ analysis is selected with the Field Stresses method. The entire model, including the yet to be excavated circular opening, are assigned elastic material. This analysis method allows a user defined constant stress field (see Define→Field Stresses in the GeoStudio menu) to be applied to the model. Field stresses are applied to the model through Draw→Field Stresses. Areas where field stresses have been applied are shown cross-hatched in red. X-extent model boundaries are fixed in the x-direction and y-extent model boundaries are fixed in the y-direction.

The Field Stress analysis ignores gravitational effects and unit weight of defined materials is ignored. In this model, because of fixities surrounding our entire model and the use of a simple stress field without any shear stresses, the model will be in equilibrium. As a result solving this step does not require iterations. In more advanced analyses, e.g., when using a plastic material, the model may not be in equilibrium for a number of reasons, such as field stress magnitudes exceed material strength or inappropriate boundary conditions. In cases where plastic material is used, it is advisable to run a Stress Redistribution analysis using the Stress Correction method immediately following the In Situ Field Stress Analysis. The Stress Correction analysis should be checked for yielding conditions. It is advisable to start from a well defined stress state with field stresses, where the model remains in the elastic domain.

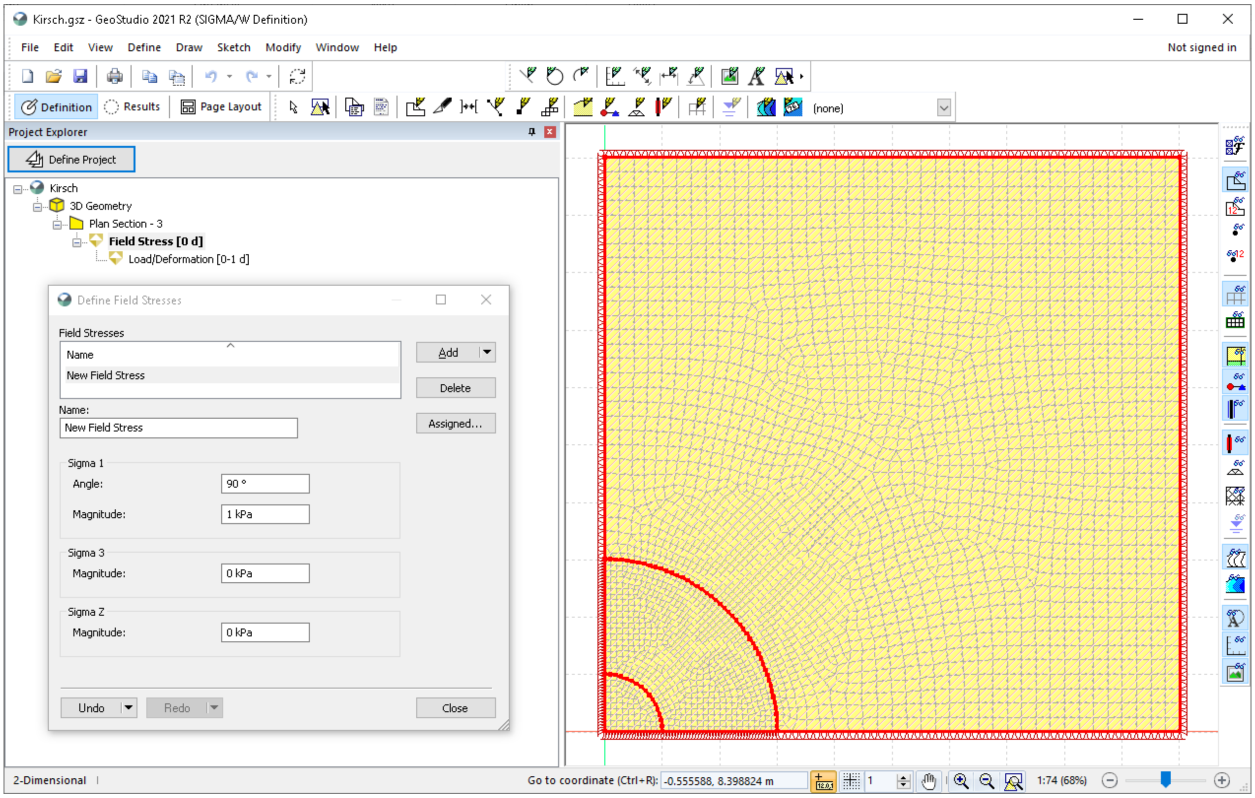

Figure 4. Field Stress definition, model mesh, and boundary conditions for In Situ analysis.

A Load/Deformation analysis is run after the Field Stress analysis, Figure 5 . The material in the circular opening is removed (the elastic material is unassigned) prior to solving. The elastic material deforms according to the assigned field stresses.

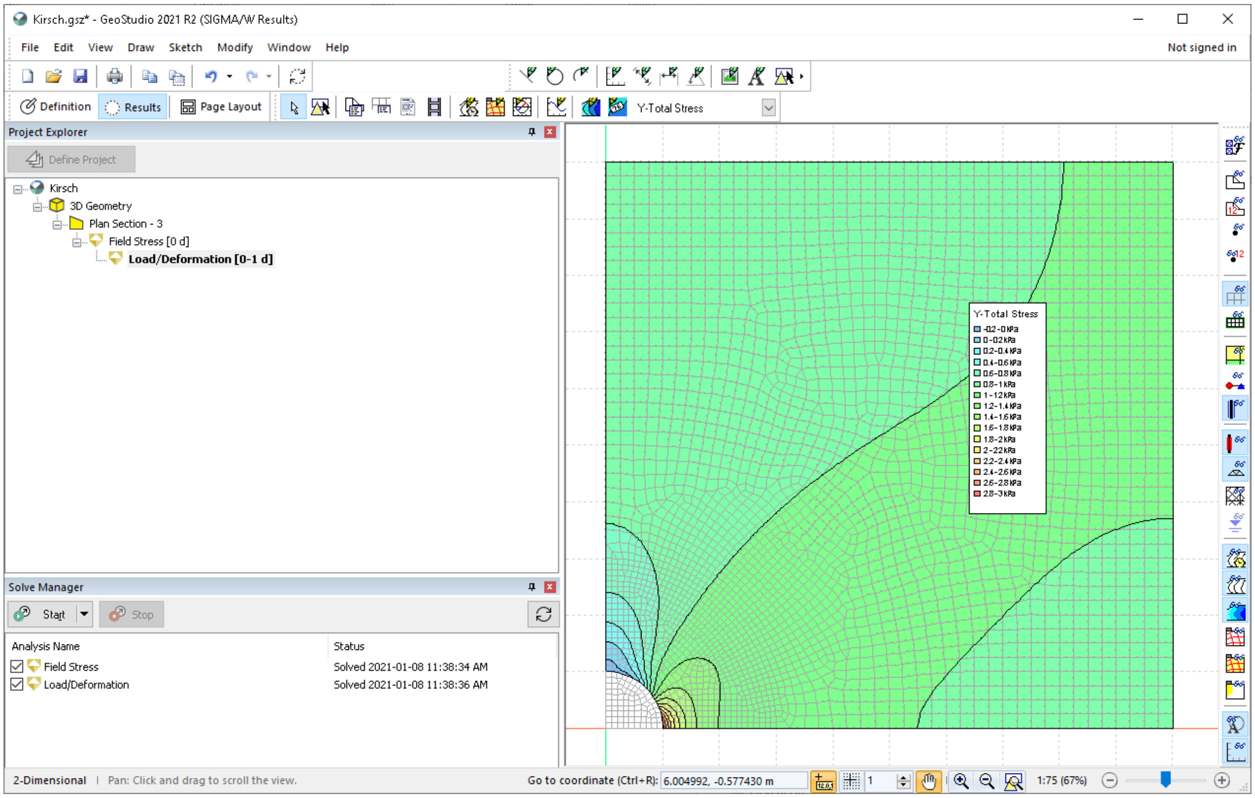

Figure 5. Load/Deformation analysis.

Results and Discussion

Displacements and stresses along the perimeter of the circular opening were extracted from the GeoStudio model and processed by a Python script to compare with analytical results (Figure 6 and Figure 7). Similar results are shown for positions along the x-axis of the model (Figure 8 and Figure 9). In these graphs, displacements are normalized to the hole radius a and stresses are normalized to the applied stress p. The match with analytical results is good and the error can be made arbitrarily small by using a finer discretization and moving the model boundaries further out.

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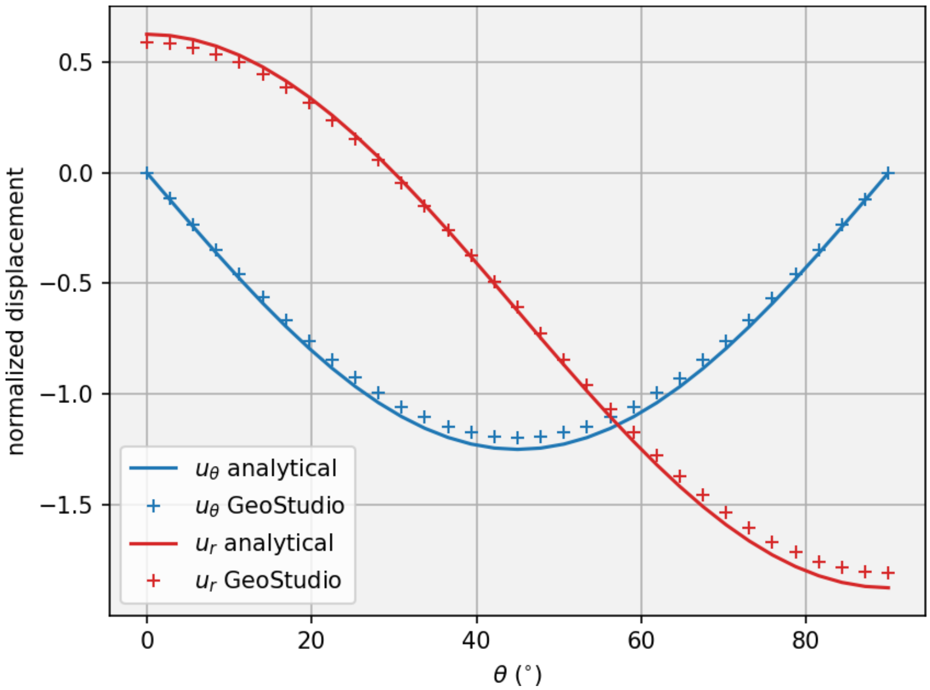

Figure 6. Radial and tangential displacement along circular perimeter versus angle.

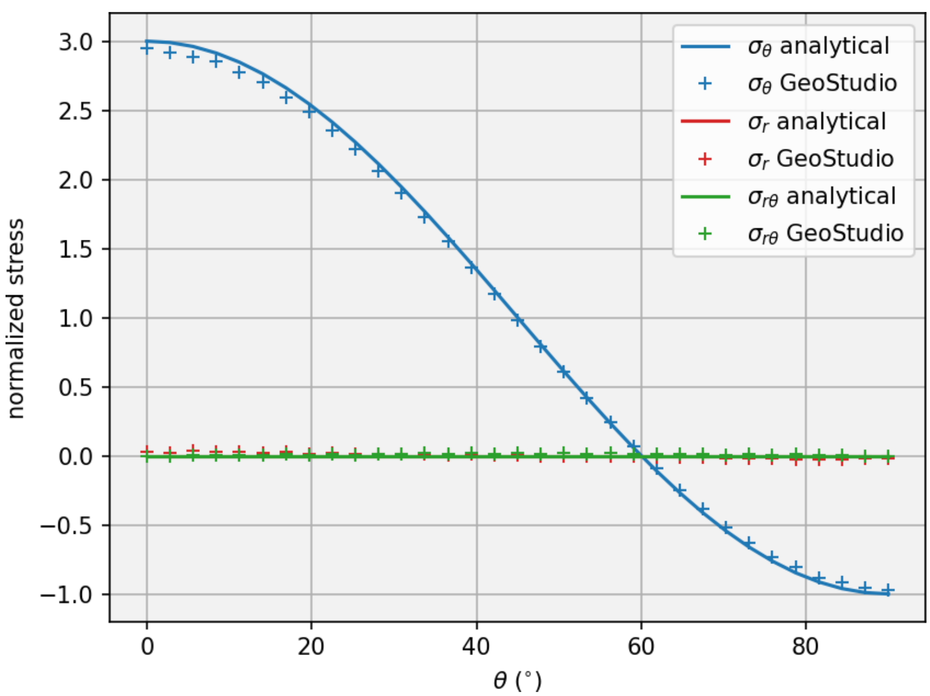

Figure 7. Radial and tangential stresses along circular perimeter versus angle.

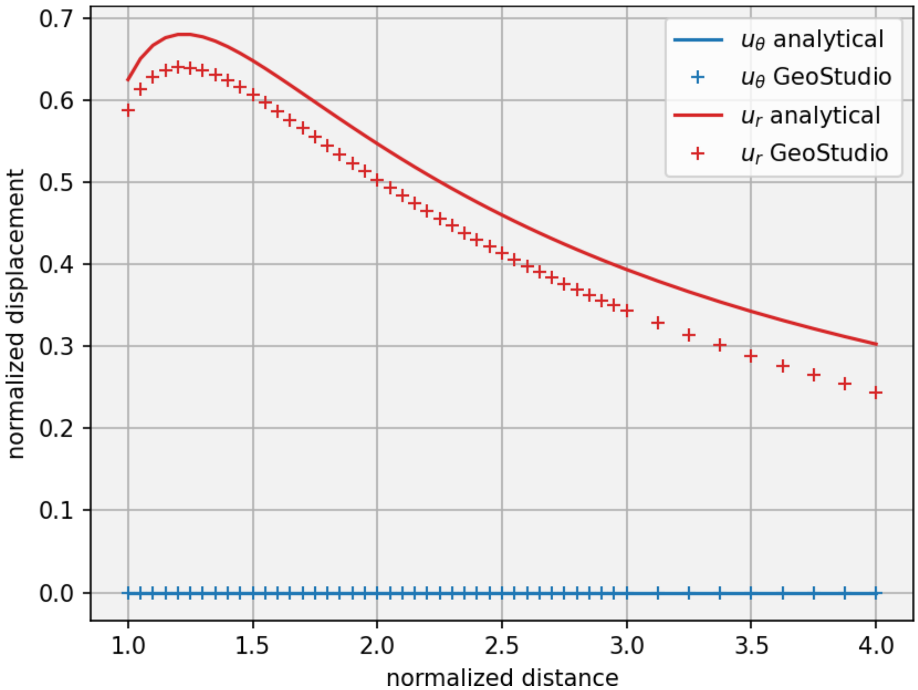

Figure 8. Radial and tangential displacement versus distance along x-axis of the model.

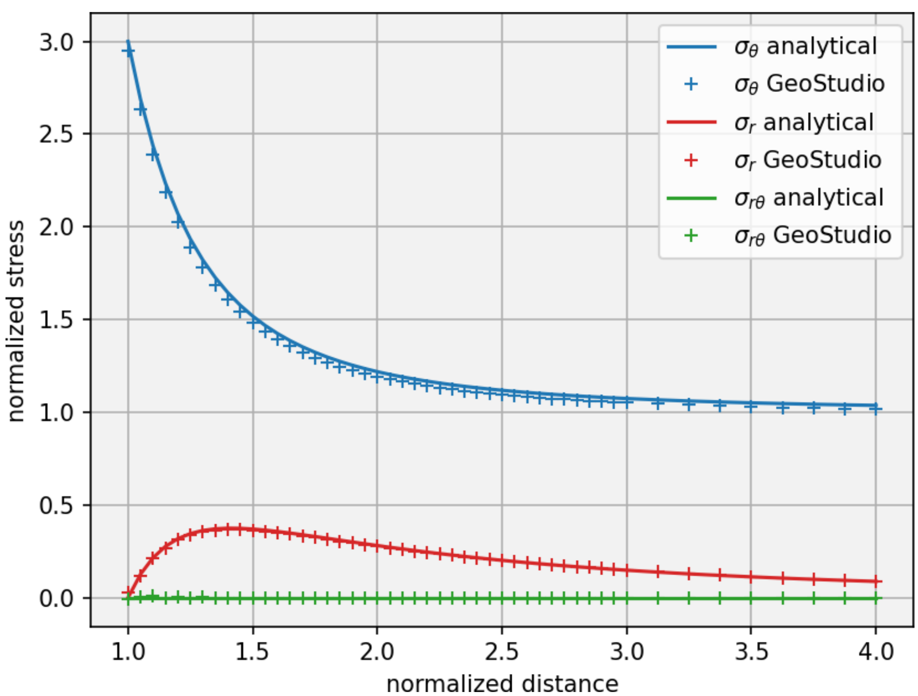

Figure 9. Radial and tangential stresses versus distance along x-axis of the model.

References

Kirsch, G. (1898) Die theorie der elastizitat und die bed¨ urfnisse der festigkeitslehre. Veit. Ver. Deut. Ing., 42: 797–807.

Brady, B. H., and Brown, E. T. (2006). Rock mechanics: For underground mining. Dordrecht: Springer. doi:10.1007/978-1-4020-2116-9