



GeoStudio Example File Finite Element Stress Based Stability

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Introduction

Though the limit equilibrium formulation is a commonly accepted method for determining slope stability, its formulation has some limitations. These limitations are primarily because it does not include a constitutive stress-strain law and therefore, does not address displacements or strains within the domain. These shortcomings can be overcome by using stresses computed in a finite element program (e.g., SIGMA/W) in a stability analysis. This example demonstrates the use of FE stresses in SLOPE/W and compares the results to a LE stability analyses to illustrate the implications.

Background

The limit equilibrium method uses static equilibrium equations to determine the factor of safety of a given slip surface, discretized into slices. This formulation seeks to find the forces acting on each slice, such that all slices are in force equilibrium and have the same factor of safety (Krahn, 2003). Consequently, the strength of each slice is reduced by the same factor to bring the system into a state of limiting equilibrium. The generated stress distribution along a slip surface is not necessarily realistic, as the limit equilibrium method does not incorporate constitutive stress-strain relationships when computing factor of safety.

Finite element analyses like SIGMA/W compute the stress distribution within the ground given the specified constitutive stress-strain relationship(s) and applied loads (including gravity). These results can be used in SLOPE/W to determine safety factors along slip surfaces, with the *SIGMA/W Stress* analysis type. First of all, the stresses ($\sigma_x, \sigma_y, \tau_{xy}$) computed at the Gauss integration points in SIGMA/W are used to determine the average stresses at the element nodes. Then, the slip surface is superimposed on the SIGMA/W results and the potential sliding mass is discretized into slices (Figure 1). For each slip surface slice, the mid-point of the slice base is determined – represented by coordinate (r, s) – and the element encompassing the base mid-point is found. The stresses at this point, $\{\sigma_{r,s}\}$, are computed given the stresses at the element nodes, $\{\sigma_{node}\}$, by:

$$\{\sigma_{r,s}\} = [N]\{\sigma_{node}\} \quad \text{Equation 1}$$

where $[N]$ is the matrix of interpolating functions used in the FE formulation, which are dependent on the coordinate (r, s) .

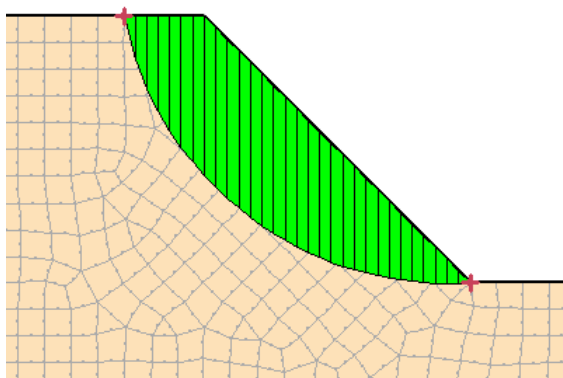


Figure 1. Slip surface superimposed on the finite element mesh.

The stresses at the base mid-point and the inclination of the slice base are used to calculate the slice base normal stress, σ_n , and mobilized shear stress, τ_m , with the ordinary Mohr-Circle

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technique. The computed shear stress multiplied by the length of the slice base gives the mobilized shear force associated with this slice. The available shear resistance, s , is determined with the slice base normal stress by:

$$s = c' + (\sigma_n - u) \tan \phi' \quad \text{Equation 2}$$

where c' is the effective cohesion, u is the pore-water pressure, and ϕ' is the effective friction angle. The available shear resistance is multiplied by the slice base length to produce the resisting shear force.

The safety factor (FS) for slice i is determined by:

$$FS_i = \frac{l_i s_i}{l_i \tau_{mi}} = \frac{s_i}{\tau_{mi}} \quad \text{Equation 3}$$

where l is the length of the slice base. Thus, each slip surface slice has a different safety factor. The overall safety factor for a slip surface is determined by integrating the shear resistance and mobilized shear along the entire slip surface with:

$$FS = \frac{\sum_{i=0}^n s_i}{\sum_{i=0}^n \tau_{mi}} \quad \text{Equation 4}$$

where n is the number of slices comprising the slip surface. As with the factor of safety in a limit equilibrium analysis, the numerator represents the resisting forces and the denominator represents the driving or de-stabilizing forces. This expression is similar to those presented by Kulhawy (1969) and Naylor (1982), and contrary to the limit equilibrium formulation, the finite element method does not require an iterative procedure to determine the safety factor.

Numerical Simulation

This example includes three analyses (Figure 2). Analysis 1a is an *Insitu* SIGMA/W analysis that generates the stresses throughout the domain. The child of this analysis, Analysis 1b, is a *SIGMA/W Stress* SLOPE/W analysis, which uses the finite element stresses generated by its parent analysis to compute safety factors as described above. Analysis 2 is a conventional limit equilibrium SLOPE/W analysis using the Morgenstern-Price formulation. This analysis was included to evaluate the variability of the results when considering finite element stresses as opposed to the typical limit equilibrium method for slope stability.

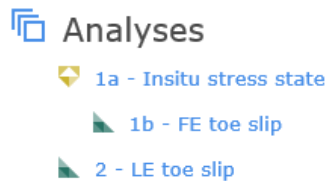


Figure 2. Analysis Tree for the project.

The project domain is a 10 m high, 1:1 slope comprised of a single material (Figure 3). For comparison purposes, the Entry and Exit method defines only 10 slip surfaces (given 10 radius

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increments), with the entry point at coordinate (22,22) and the exit point at the toe (35,12). This slip surface definition is the same in both the finite element and limit equilibrium SLOPE/W analyses. No pore water pressure conditions are defined in the analyses.

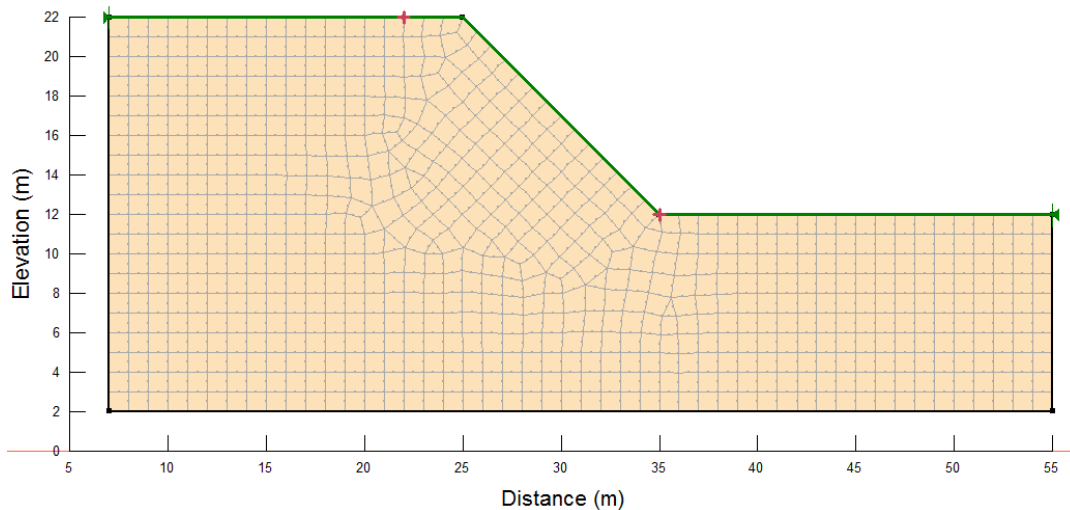


Figure 3. Example configuration and finite element mesh.

The material properties in the SIGMA/W analysis are defined with the Linear Elastic material model, given total stress parameters as no pore water pressures are defined. The total E-modulus is set to 10,000 kPa, with a soil unit weight of 20 kN/m³, and a Poisson's ratio of 0.3334. The left and right domain boundaries are fixed in the x-direction, while the bottom boundary is fixed in both the x and y directions. The same material is applied in the two SLOPE/W analyses (1b and 2). The Mohr-Coulomb material model defines the material properties with a cohesion of 15 kPa, friction angle of 28°, and unit weight of 20 kN/m³.

Results and Discussion

The results from Analysis 1a can be viewed in many different ways; for example, contour plots can be created to visualize various parameter values throughout the domain (Figures 4 and 5). The generated vertical stress contours are evenly spaced along the left and right sides of the domain (Figure 4). This indicates that vertical stress in these locations is directly related to the overburden stress, while the x-y shear stress is essentially zero. This is not the case under the slope, particularly in the toe area, as the vertical stress contours are not parallel to the ground surface. Thus, the shear stress concentration in the toe area (Figure 5) affects the vertical stress contours (Figure 4). These conditions affect the factor of safety.

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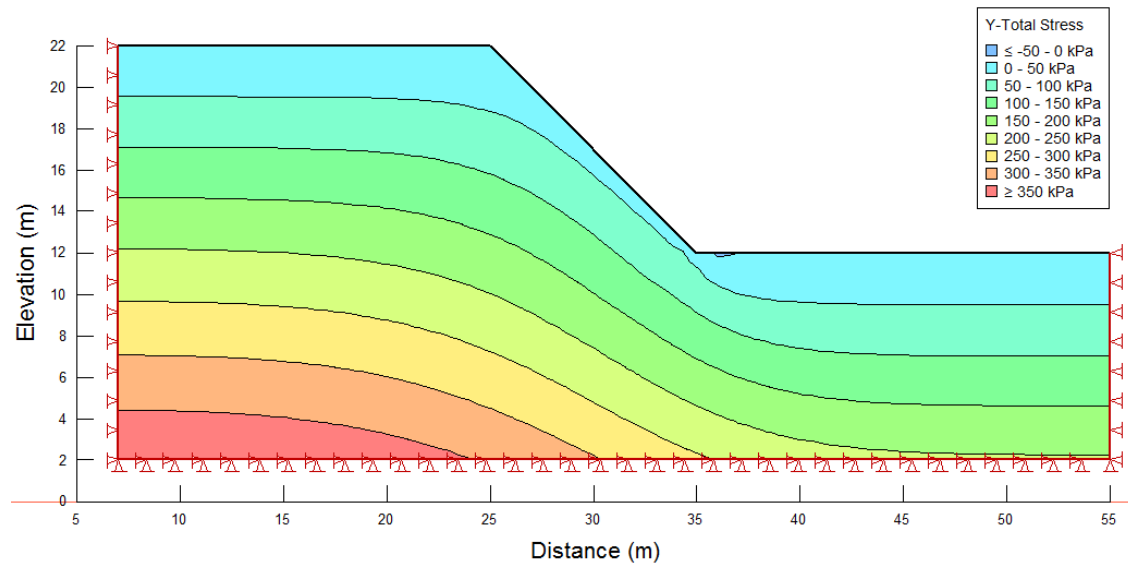


Figure 4. Vertical stress contours generated by Analysis 1a.

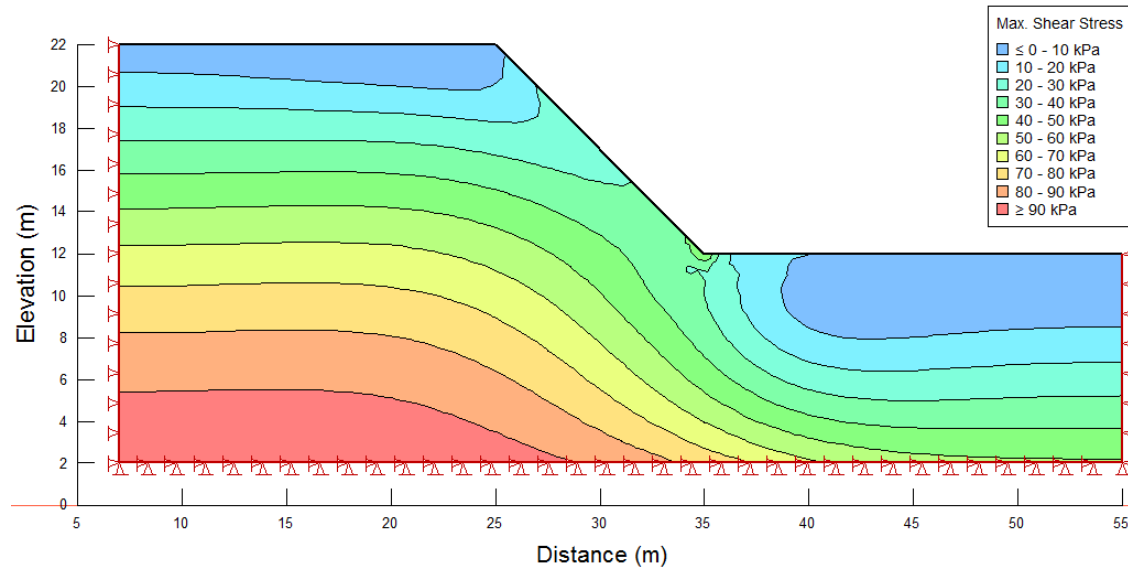


Figure 5. Maximum shear stress contours from Analysis 1a.

The state of stress can also be inspected at any node or Gauss integration region with the Draw Mohr Circles command (Figure 6). When σ_x , σ_y and τ_{xy} are known at a point, the shear and normal stress on any other plane in space at that point is determined with the Mohr-Circle technique. The directions of the principal stresses are shown (Figure 6) but they are not directly used in the safety factor calculations.

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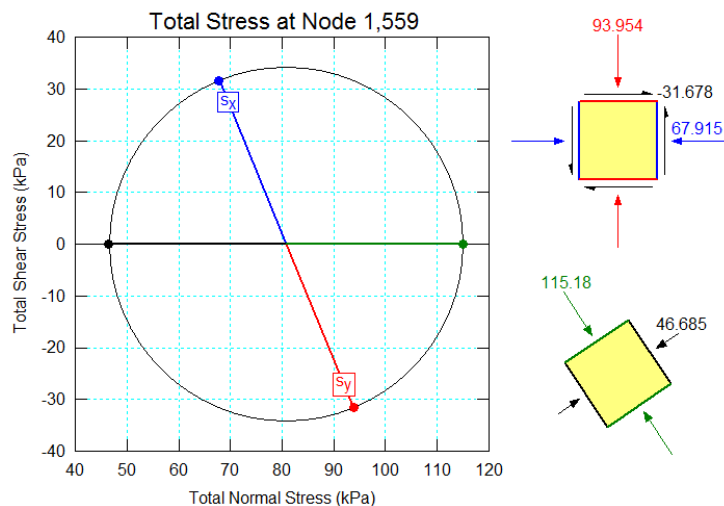


Figure 6. A typical Mohr-Coulomb diagram and associated space diagram generated at a node in Analysis 1a.

The safety factor calculated for each trial slip surface using the finite element stresses can be illustrated with a slip surface color map, similar to limit equilibrium analyses (Figure 7). The slip surface with the lowest safety factor (of 1.431) is near the middle of the trial slip surfaces, indicated by the white slip surface (Figure 7). Overall, the safety factor was relatively similar for the ten slip surfaces.

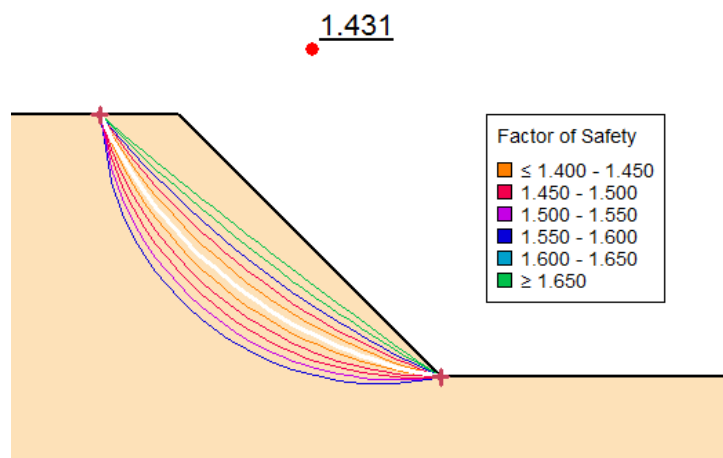


Figure 7. Stability analysis results based on the finite element stresses (Analysis 1b).

The Draw Graph feature offers additional interpretation capabilities. For example, the local safety factors, calculated by Equation 3, can be plotted over slice number (Figure 8). The two components used to calculate the local safety factor, shear resistance and mobilized shear, are plotted in Figure 9. At the crest (slice 1), the mobilized shear is very small relative to the available shear resistance. Consequently, the computed value for the safety factor is very large (off the graph). At the toe, the mobilized shear is greater than the shear resistance (Figure 9) so the local factor of safety is less than 1.0 (Figure 8). These low safety factors are a result of the Linear-Elastic constitutive model used in SIGMA/W to establish the *in situ* stress state. In order to obtain reliable stability factors when using finite element stresses, the initial SIGMA/W model must produce reasonable *in situ* stress states. This is not always an easy task, especially if the geologic history of the ground plays an important role (e.g., high K_0 conditions).

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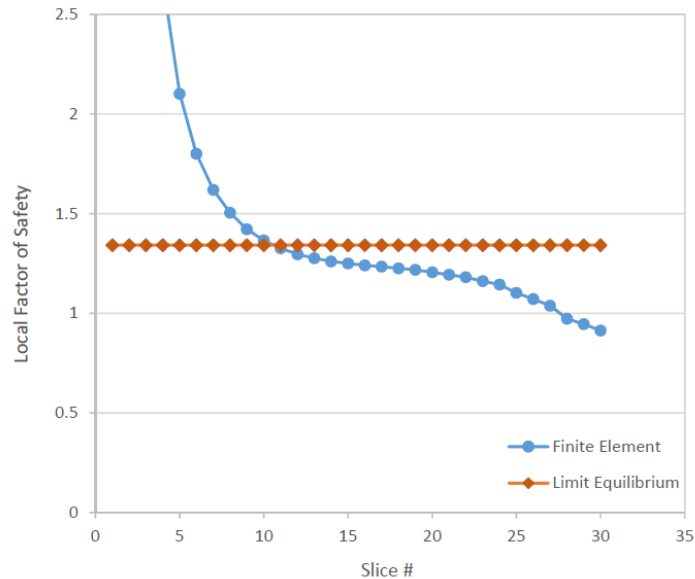


Figure 8. Local safety factors for the critical slip surface from the finite element (Analysis 1b) and limit equilibrium (Analysis 2) methods.

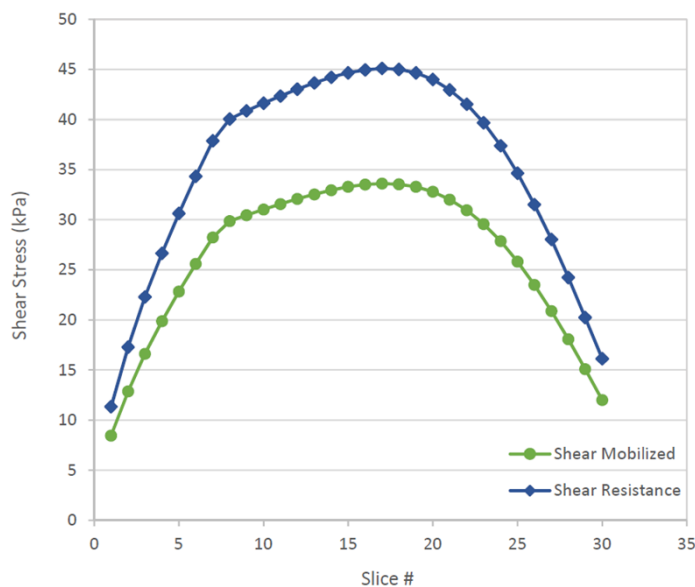


Figure 9. Mobilized shear and shear resistance along the slip surface.

The global safety factor (1.431; Figure 7) is not the average of the local safety factors. It is a ratio of the total available shear resistance along the slip surface to the total mobilized shear. In other words, the global safety factor is the area under the shear resistance curve (in Figure 9) divided by the area under the mobilized shear curve.

The Morgenstern and Price limit equilibrium method (Analysis 2) found the same critical slip surface as the finite element stability analysis (Analysis 1b). The factor of safety determined in Analysis 2 is 1.342 (Figure 10), which is slightly lower than the global safety factor from the finite element stability analysis (Figure 7). The factor of safety differs in the two analyses because the normal stresses acting on the critical slip surfaces are different (Figure 11). However, the difference in area under the two normal stress curves is small, which explains the relatively similar results generated by the finite element and limit equilibrium methods.

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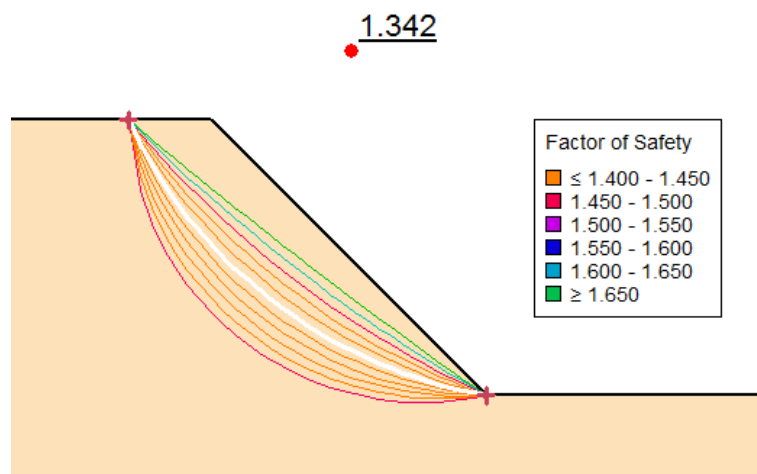


Figure 10. Stability results from the Morgenstern and Price limit equilibrium analysis (Analysis 2).

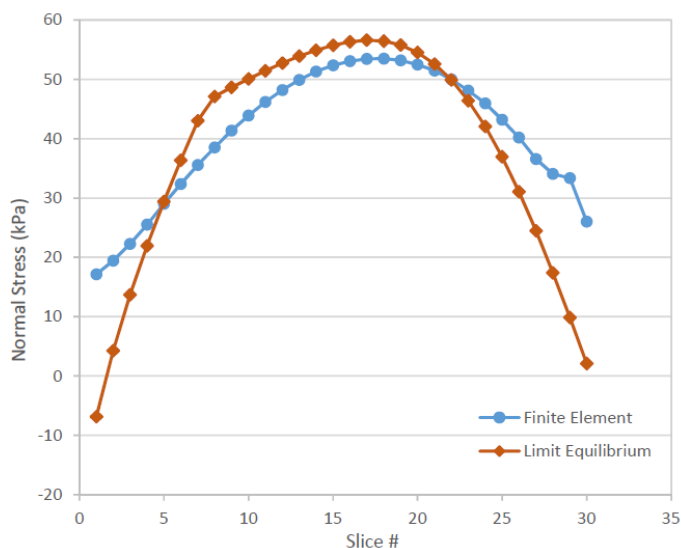


Figure 11. Normal stress distributions along the critical slip surface from the finite element (Analysis 1b) and limit equilibrium (Analysis 2) methods.

As previously mentioned, the limit equilibrium formulation finds the forces acting on each slice, such that the factor of safety is the same for each slice (Figure 8). These requirements result in stresses that are not necessarily representative of the actual stresses in the ground. A comparison of the normal stresses generated by Analyses 1b and 2 (Figure 11) demonstrates that the limit equilibrium stresses do not match the *in situ* stresses from the finite element analysis. This is particularly true if there are shear stress concentrations in the ground, as observed near the slope toe and as manifest by reinforcement within the domain (e.g., tie-back anchors).

Summary and Conclusions

This example demonstrates the use of finite element stresses in stability analyses to overcome limit equilibrium shortcomings. When the *in situ* stresses are considered, the local safety factors vary over the slip surface based on the stress conditions at the base of each slice. In addition, an iterative procedure is not required to compute the factor of safety. However, the finite

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element method requires that reasonable *in situ* stresses are first generated in a SIGMA/W analysis.

Fortunately, the limit equilibrium method is generally reliable for engineering practice in spite of its limitations. The results from limit equilibrium and finite element stability analyses are reasonably close if the normal stress distribution along the slip surface is primarily related to the overburden stress, as is the case for relatively flat, natural slopes. Greater differences arise when shear stress concentrations are present, as observed at the slope toe (due to the slope steepness). Projects with soil-structure interaction (i.e., reinforcement) typically have high shear stress concentrations. Thus, the finite element approach is useful for analyzing the stability of man-made structures; whereas, the limit equilibrium approach is better for analyzing the stability of natural slopes, especially when geologic history plays a significant role in the stress state.

References

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