



GeoStudio Example File Limit Equilibrium Formulation

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Introduction

SLOPE/W is based on what is called a Limit Equilibrium (LE) formulation. Fundamentally, the objective in a LE formulation is to find the point or condition where all of the driving or destabilizing force is equal to all of the resisting or stabilizing forces. This condition is determined by satisfying all equations of statics. That is, the summation of forces in the horizontal (F_x) and vertical (F_y) directions must equal zero and the summation of moments (M_0) about any point must equal zero. In equation form:

$$\sum F_x = 0 \quad \text{Equation 1}$$

$$\sum F_y = 0 \quad \text{Equation 2}$$

$$\sum M_0 = 0 \quad \text{Equation 3}$$

In order to achieve this, it is necessary, through an iterative procedure, to find a number by which the shear strength of the soil must be reduced. This number is called the factor of safety.

The equation of statics is the extent of the physics underlying a LE formulation. No stress-strain constitutive relationship, for example, is invoked. This creates some limitations in the LE formulation. To make effective use of a LE stability formulation, it is essential to recognize and comprehend the fundamentals and consequences of the formulation. This SLOPE/W document highlights the essence of the SLOPE/W LE formulation, draws attention to the resulting limitations and provides meaning to the end results.

Background

SLOPE/W is formulated on the basis of two factors of safety equations: one with respect to moment equilibrium (F_m) and one with respect to horizontal force equilibrium (F_f). A simplified form of the two equations is as follows (the complete equations are given in the SLOPE/W Engineering Book).

$$F_m = \frac{\sum [c' l + (N - ul) \tan \phi']}{\sum W \sin \alpha} \quad \text{Equation 4}$$

$$F_f = \frac{[c' l \cos \alpha + (N - ul) \tan \phi' \cos \alpha]}{\sum N \sin \alpha} \quad \text{Equation 5}$$

where the variables c' and ϕ' are material properties (effective cohesion and effective angle of friction), l and α are geometric properties, u is the pore-water pressure and W is the slice weight. These variables are all well-defined. It is the slice base normal (N) that is undefined and needs to be determined.

The normal force N is derived from the summation of forces in the vertical direction on each slice. The resulting equation is:

$$N = \frac{W + (X_R - X_L) - \left[\frac{c' l \sin \alpha - \tan \phi' \sin \alpha}{F} \right]}{\cos \alpha + \frac{\sin \alpha \tan \phi'}{F}} \quad \text{Equation 6}$$

It is intuitive that N should depend on the slice weight (W), the inter-slice shear ($X_R - X_L$), and the base inclination (α). It is perplexing, however, that the base normal N is dependent on the factor of safety F .

What this equation is inferring is that the state of stress in the ground is a function of the factor of safety. From the theory of solid mechanics, it is known that this is not the case. This is the first clue that a LE formulation has some limitations and is based on some simplifying assumptions.

By substituting the N equation into the F_m and F_f equations, F appears on both sides of the factor of safety equations. This makes the equations non-linear and, consequently, an iterative procedure is required to find a solution.

When the inter-slice shear forces are ignored, F_m is the Bishop Simplified factor of safety and F_f is the Janbu Simplified factor of safety. For the more rigorous methods, such as the Spencer and Morgenstern-Price (M-P) methods, that include the inter-slice shear, it is necessary to find a scaling factor λ in the following equation so that F_m and F_f are the same:

$$X = E\lambda f(x) \quad \text{Equation 3}$$

where X is the inter-slice shear, E is the inter-slice normal, $f(x)$ is a shape distribution function and λ is the factor used to scale the distribution shape function.

The Spencer Method is based on a constant or straight-line horizontal $f(x)$. This results in a constant X/E ratio from crest to toe along the slip surface. With a constant $f(x)$, and when F_m equals F_f , the factor of safety, by definition, is the Spencer factor of safety. The M-P solution in SLOPE/W is identical to the Spencer solution, except that $f(x)$ can be some other form. The default in SLOPE/W is a half-sine function.

The solution scheme in SLOPE/W follows three different stages:

1. In Stage 1, SLOPE/W computes the Ordinary factor of safety. Since all inter-slice forces are ignored in the Method and the factor of safety equation is linear, the factor of safety can be computed directly; that is, no iterative procedure is required.
2. The factor of safety from Stage 1 is used to seed the iterative procedure for computing the Bishop Simplified and Janbu Simplified safety factors. From there on, the procedure follows a repeated substitution scheme. The most recently computed factor of safety is used for the next subsequent iteration. This continues until the two most recently computed safety factors are within a specified tolerance.
3. The third stage involves computing F_m and F_f for a range of trial lambda λ values. These results are used to create a plot such as in Figure 1. The curve with the red, closed-square symbols is the factor of safety with respect to moment equilibrium; that is, F_m . The curve with the blue, open-squares is the factor of safety with respect to horizontal force equilibrium; that is, F_f .

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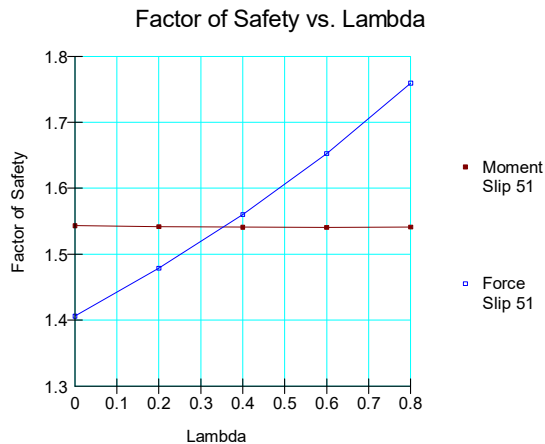


Figure 1. Typical FofS versus lambda plot.

Where the two curves cross is the Spencer or M-P factor of safety. From this graph, it is evident that λ for this particular case needs to be 0.35 for F_m and F_f to be the same. At this point, the solution satisfies both overall moment equilibrium and overall horizontal force equilibrium.

The Bishop method only satisfies moment equilibrium and ignores inter-slice shear forces. The Bishop factor of safety, therefore, falls on the F_m curve where lambda is zero. A lambda value of zero means the inter-slice shear forces are zero. The Janbu Simplified method only satisfies horizontal force equilibrium, but also ignores inter-slice shear forces. The Janbu factor of safety, therefore, falls on the F_f curve where lambda is zero.

With this scheme, all of the safety factors representing various methods can be determined. This removes the need for special schemes for each of the various methods. The graph shown in Figure 1 is created for each and every trial slip surface when the Spencer and M-P Methods are selected.

A Limit Equilibrium formulation such as in SLOPE/W has two consequences (Krahn, 2003):

1. The iterative procedure continues until the forces acting on each slice are such that the factor of safety is the same for each slice; and
2. The iterative procedure continues until the forces acting on each slice are such that the slice is in force equilibrium.

Consider, for example, the slice free-body diagram and force polygon in Figure 2. The closure of the force polygon indicates the slice is in force equilibrium.

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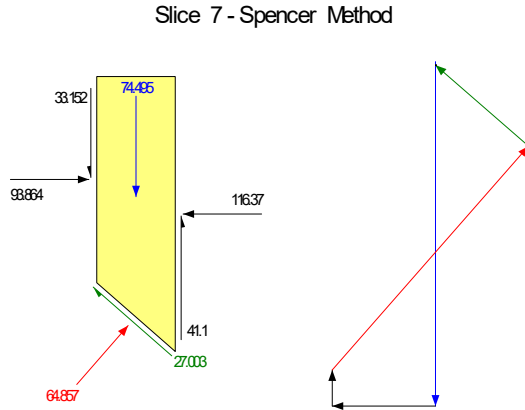


Figure 2. Force polygon showing the slice force equilibrium.

Insisting that the factor of safety must be the same for each and every slice has the unfortunate consequence that the stresses along the slip surface do not necessarily represent the actual stresses in the field. In reality, the local factor of safety for each slice is not constant, especially if concentrated loads are present. This, however, is one of the inherent assumptions in a LE formulation, which is one of the limitations.

Another important point is that the inter-slice forces are not related to the soil strength and pore-water pressures internal to the potential sliding mass. Once again, they are simply the forces required to meet the above two conditions.

SLOPE/W does not seek to satisfy moment equilibrium of each individual slice. Attempting to satisfy moment equilibrium of each slice requires imposing some kind of stress distribution on the slip surface. Doing so over constrains the problem, making it impossible in all but the simplest cases to obtain a solution. SLOPE/W satisfies overall moment equilibrium, but not individual slice moment equilibrium.

Only total forces can be used in resolving the equilibrium equations. Water forces arising from pore-water pressures internal to the sliding mass do not come into play in the slice equilibrium. The forces on the free-body diagram in Figure 2 are total forces – there are no forces specifically representing water forces. The pore-water pressure only comes into play in the slice base shear resistance when computing the available shear strength.

The primary reason for the limitations in the LE formulation is that it does not satisfy strain and/or displacement compatibility. Or, in other words, the formulation does not include a constitutive stress-strain law.

Numerical Simulation

The relationship amongst the safety factors from the various Methods depends heavily on the slip surface shape. An example file was created to illustrate the influence of slip surface shape. Two analyses are included in the example file (.GSZ) to illustrate the use of the circular slip surface and planar slip surface (Figure 3). The model domain consists of a single material with a 1:1 slope (Figure 4). The Mohr-Coulomb material model is used to describe the soil, with a unit weight of 20 kN/m³, cohesion (c) of 10 kPa and friction angle (ϕ) of 26°. The pore-water pressure conditions are defined using a piezometric line.

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Analyses

- 1 - Circular Slip Surface
- 2 - Planar Slip Surface

Figure 3. Analysis Tree for the Project.

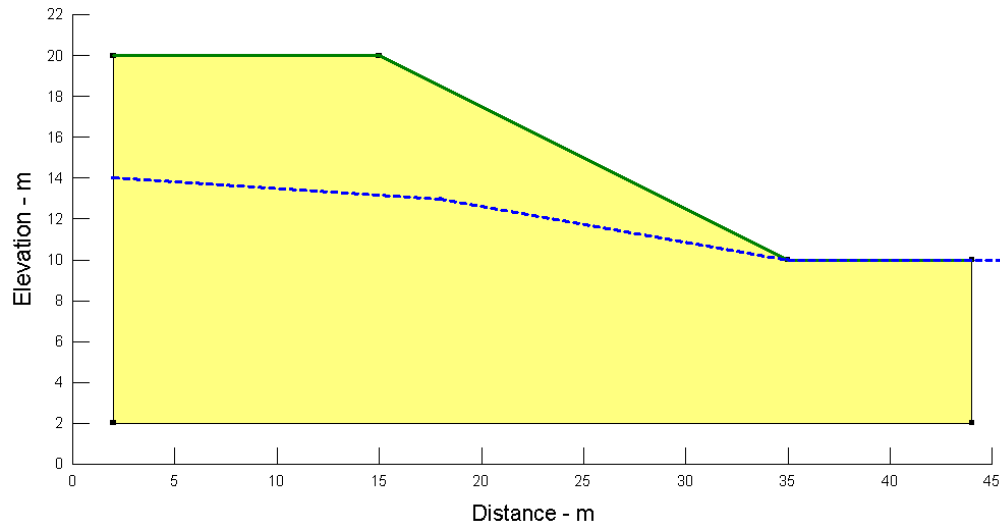


Figure 4. Problem configuration.

In the first analysis, the slip surface is defined using the Entry-Exit option (Figure 5). In the second analysis, the Fully Specified option was used to define a planar slip surface (Figure 6). This option allows the user to specify the slip surface with a series of points. In this case, only two points were selected to create a single line. A third case is also presented below to illustrate the behavior of composite slip surface, but there is no analysis for this illustration in the example file.

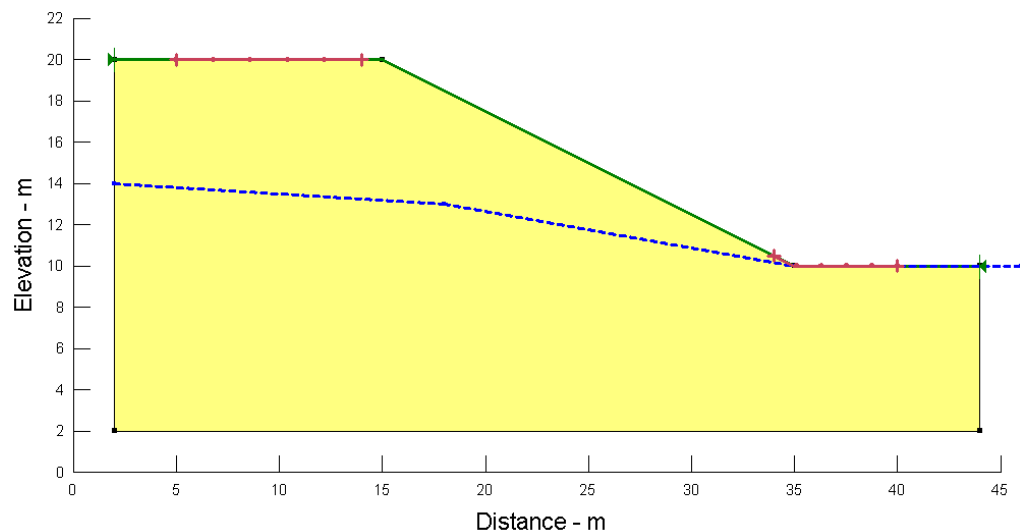


Figure 5. Location of the left and right surface portions used to illustrate the circular slip surface.

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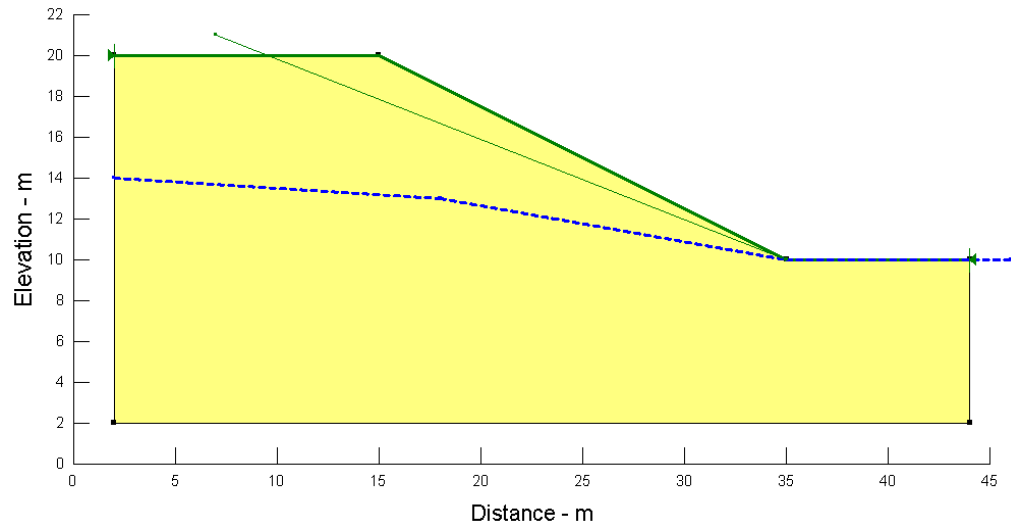


Figure 6. Location of the planar slip surface using the Fully Specified slip surface option.

Results and Discussion

In the case of a curved slip represented by the arc of a circle, the Factor of Safety and critical slip surface are shown in Figure 7. Notice that the factor of safety with respect to moment equilibrium is not influenced by the inter-slice shear; that is, the curve is essentially flat (Figure 8). As a result, the Spencer factor of safety (cross-over point) is the same as the Bishop factor of safety (moment curve where lambda is zero).

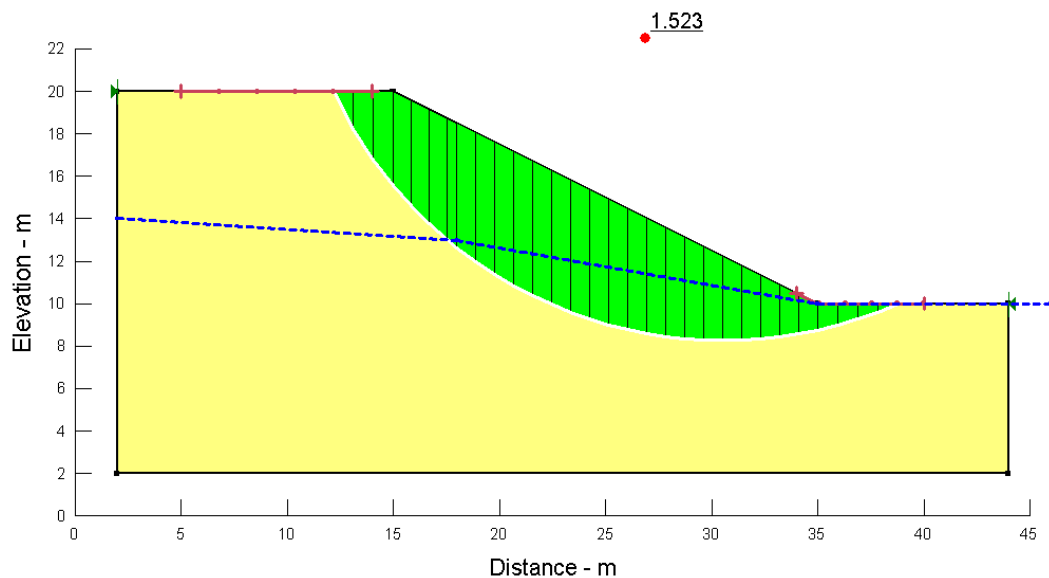


Figure 7. Factor of Safety for the circular slip surface.

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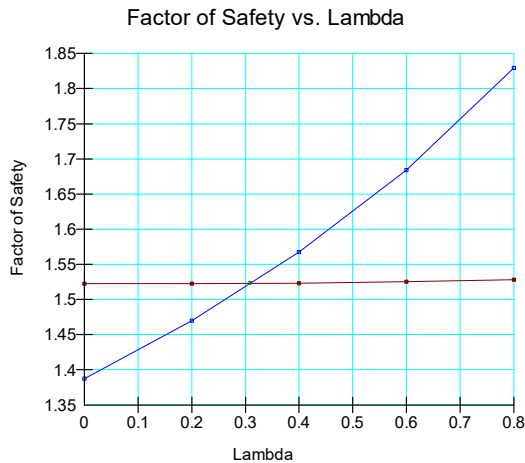


Figure 8. Factor of Safety vs. Lambda for the circular slip surface analysis.

The reason for this is that the sliding mass can rotate without distortion or without any slippage between the slices. This is not true for horizontal translation and, therefore, the factor of safety with respect to force equilibrium is strongly influenced by the inter-slice shear. In this case, the Bishop and Spencer safety factors are the same, while the Janbu Simplified safety factor is significantly lower.

The Factor of Safety for the planar slip surface is shown in Figure 9. Now the F_m and F_f curves have flipped positions (Figure 10). Now the factor of safety with respect to force equilibrium is independent of the inter-slice shear, while the factor of safety with respect to moment equilibrium is highly dependent on the inter-slice shear.

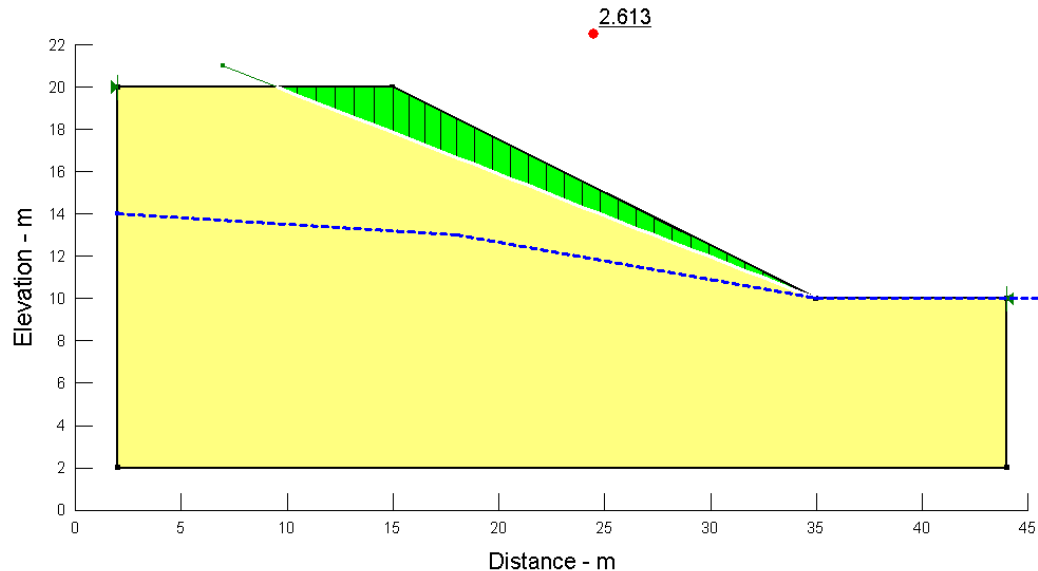


Figure 9. Factor of Safety of the planar slip surface.

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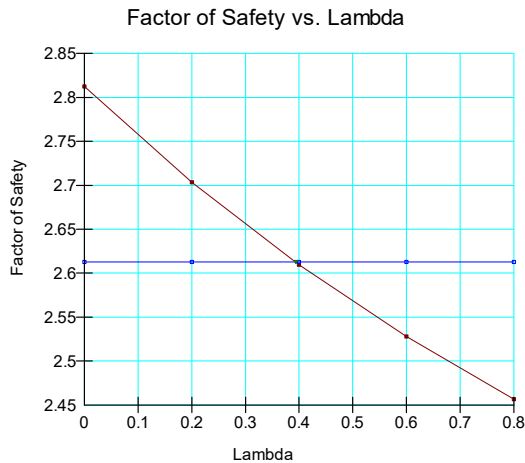


Figure 10. Factor of Safety vs. Lambda for the planar slip surface.

In this case, lateral translation can occur without distortion in the sliding mass. Rotation, however, would require large distortion or slippage between the slices. In this case, the Janbu and Spencer safety factors are the same, while the Bishop safety factor is significantly higher.

For a composite slip surface (Figure 11), both the F_m and F_f curves are sensitive to the inter-slice shear (λ). In this case, the M-P factor of safety (cross-over point) falls between the Bishop and Janbu factors of safety.

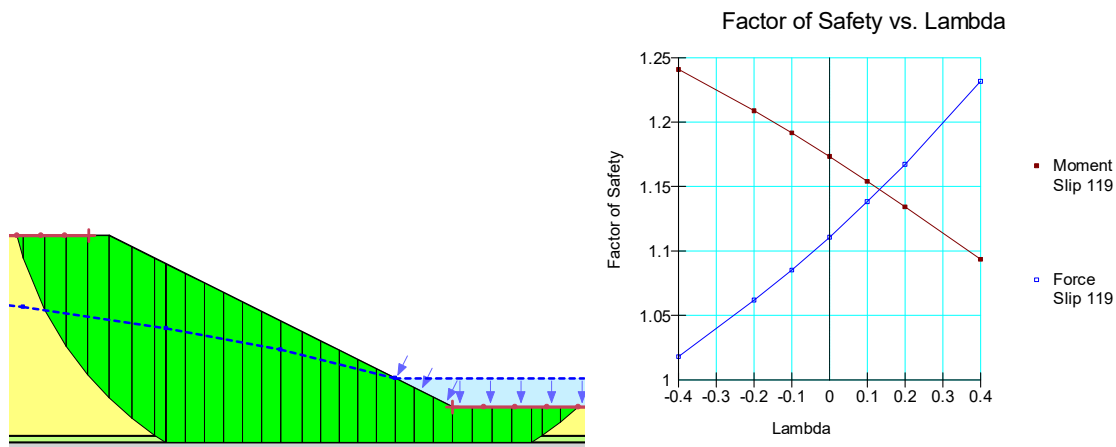


Figure 11. Behavior of a composite slip surface.

These illustrations show why it is necessary to always rely on a rigorous method such as M-P or Spencer that satisfies both moments. For a circular slip surface, the Janbu factor of safety under-estimates the actual margin of safety. For a planar slip, the Bishop factor of safety over-estimates the actual margin of safety.

Furthermore, these illustrations show that it is not possible to say that a simple method always errors on the safe side. In both Figure 10 and Figure 11, it is evident the simpler Bishop factor of safety over-estimates the more rigorous M-P or Spencer factor of safety.

There are consequences in the Limit Equilibrium formulation. Unfortunately, it is not always possible to achieve the required conditions; that is, it is not always possible to find the forces on each slice such that the factor of safety is the same for each slice. In numerical terminology, this manifests itself in non-convergence or it is not possible to obtain a converged solution.

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Lack of convergence usually happens when there are large shear-stress concentrations on the slip surface arising from either very sharp corners in the slip surface shape or concentrated loads arising from reinforcement. Consider the case of a sliding stability analysis of a gravity retaining wall (Figure 12).

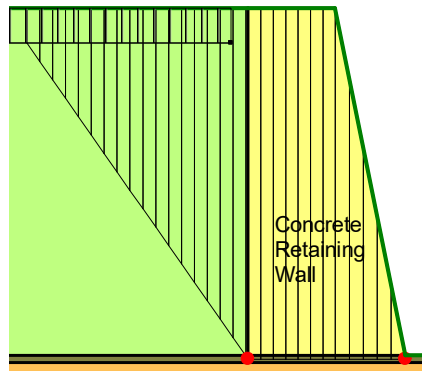


Figure 12. Illustration of a gravity wall sliding analysis.

When the trial slip surface becomes too steep and the internal angle becomes too sharp, the FofS versus Lambda plot appears as in Figure 13. It is not possible to find a cross-over point. The specified lambda range extended up to 0.8, but at a lambda value of 0.8 it was not possible to compute one or the other of the F_m and F_f safety factors. Convergence difficulties often occur when the F_m and F_f curves lean towards being parallel, as illustrated in Figure 14. It is vitally important to always inspect the FofS versus Lambda plots when evaluating the validity of stability analysis.

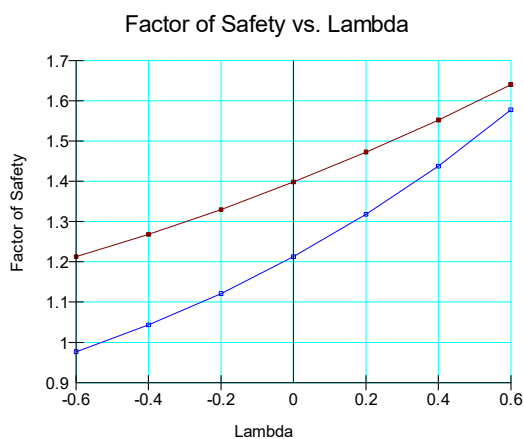


Figure 13. Illustration of a non-converged FofS versus Lambda plot.

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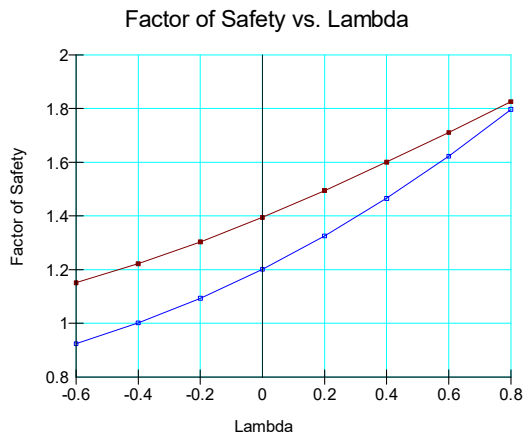


Figure 14. Illustration of the F_m and F_r tending to being parallel.

Summary and Conclusions

As already mentioned earlier, the restraints inherent in a Limit Equilibrium formulation means that the computed stress distributions along the slip surface are not necessarily representative of the actual *in situ* conditions. While this is true, it does not mean that the overall global factor of safety is invalid. Locally, the LE factor of safety may be incorrect, but globally the computed factor of safety is acceptable as a margin of safety against failure. A LE formulation, such as SLOPE/W, consequentially remains an acceptable tool for use in geotechnical engineering practice.

The FofS versus Lambda plots created by SLOPE/W for each trial slip surface are a useful tool for understanding the relationships amongst the factor of safety methods and judging the acceptability of the solution convergence. Inspection of these plots needs to be standard practice when using SLOPE/W for stability analyses.

Overcoming the limitations in a LE formulation requires adding more physics to the solution, such as a stress-strain law. This can be done by first doing a SIGMA/W finite element stress-deformation analysis and then use the FE-computed stresses in a SLOPE/W stability analysis. This is described in a companion example file called Finite Element Stresses.

References

Krahn, J. 2003. The 2001 R.M. Hardy Lecture: The limits of limit equilibrium analyses. Canadian Geotechnical Journal 40: p. 643-660.