GeoStudio Example File
Verification – Air convection in a cube

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Introduction

Convection is omnipresent in nature and plays an important role in numerous engineering and earth science problems. It occurs as pore fluid moves within the soil due to external forces (such as pressure) or internal forces (such as gravity). These latter forces act upon fluid density variations brought on by non-uniform temperature fields and/or non-uniform concentration of fluid components. In the case of a non-uniform temperature field, the fluid closest to the heat source becomes less dense and rises due to an increase in kinetic energy. This movement alters the thermal regime, which affects the fluid density, and thereby creates a coupled system. The manner in which this buoyancy-driven flow occurs depends on the boundary conditions and the geometry (or shape) of the domain.

The objective of this example is to verify the airflow formulation against a widely used benchmark for natural convection in a fluid-saturated porous cube. In this problem, the cubic enclosure is heated from below, cooled from the top, and does not allow heat transfer through its vertical walls. This exercise lends credibility to the use of the software in multi-dimensional problems involving natural convection.

Numerical Simulation

The purpose of the example is to model natural convection in a porous cube with impervious and adiabatic vertical walls and impervious and isothermal horizontal walls. The fluid is initially hydrostatic and the initial temperature is representative of a slightly disturbed conductive state. At some point, a constant temperature differential is applied between the horizontal walls. The pore-air closest to the heated wall expands and rises. This results in convective motion as the top-heavy fluid column becomes unstable. The ideal gas law adequately characterizes the changes in density that occur in the cube at low pressures, and other fluid properties are assumed to remain constant. The domain is discretized in a structured mesh of 50×50×50 hexahedral elements and an exponential time stepping scheme is used in the transient analyses. The analyses are deemed to have reached steady-state conditions when the differences between the heat fluxes at the upper and lower walls is less than 2%.

The Rayleigh-Darcy dimensionless number characterizes the flow regime within the cubic enclosure (Lapwood, 1948):

$$\begin{aligned} Ra_D &\equiv Ra Da \frac{k_{t,a}}{k_t} \\ &= \frac{g\beta_{t,o_a} H^4 \Delta T/H \, {\mu_{a_o}}/{\rho_{a_o}}}{\left({\mu_{a_o}}/{\rho_{a_o}}\right)^2 \; k_{t,a}/C_{t,a}} \, \frac{k_a \dfrac{\mu_{a_o}}{\rho_{a_o} g}}{H^2} \, \frac{k_{t,a}}{k_t} \end{aligned} \qquad \text{Equation 1}$$

where Ra, Da are the Rayleigh and Darcy dimensionless numbers, respectively; $k_{t,a}$ is the thermal conductivity of air; k_t is the thermal conductivity of the porous media; g is the gravity constant;

$\beta_{t,o_a} = -\frac{1}{\rho_{a_o}}(\partial \rho_a / \partial T)_{P,C}$ is the reference volumetric thermal expansion coefficient of air at constant pressure and concentration; H is the reference dimension, which is taken equal to the height of the cube; ΔT is the temperature differential; μ_{a_o} is the reference dynamic viscosity of air; ρ_{a_o} is the reference density of air; $C_{t,a}$ is the volumetric heat capacity of air at constant pressure and temperature; and k_a is the air conductivity of the porous media. In order to obtain results at various Rayleigh-Darcy numbers, the transient analyses are set up with different values of air conductivity and constant thermal material properties, such that:

$$k_a = \frac{\left(\mu_{a_o}/\rho_{a_o}\right)^2 \;\; k_{t,a}/C_{t,a} \;\; H^2 \;\; k_t}{g\beta_{t,o_a} H^4 \Delta T / H \mu_{a_o}/\rho_{a_o} \dfrac{\mu_{a_o}}{\rho_{a_o} g} k_{t,a}} Ra_D \qquad \text{Equation 2}$$

where μ_{a_o} = 1.812×10^{-5} N s/m^2, ρ_{a_o} = 1.225 kg/m^3, g = 9.807 m/s^2, β_{t,o_a} = 3.470×10^{-3} 1/K, H = 1 m, ΔT = 30 K = 30 °C, $k_{t,a}$ = 0.025 J/s/m/°C, $C_{t,a}$ = 1.231×10^3 J/m^3/°C, k_t = 0.984 J/s/m/°C. The influence of the flow regime is measured with the mean Nusselt dimensionless number $\bar{N}u$, which is defined as the ratio of convective to conductive heat transfer normal to the upper boundary:

$$\bar{N}u = -\frac{\frac{1}{B}\left(\int_{x=0}^{B} \frac{\partial T}{\partial y} dx\right)_{y=H}}{\Delta T / H} \qquad \text{Equation 3}$$

where B is the width of the enclosure.

Results and Discussion

Numerical studies have shown that various steady-state convective flow structures (or patterns) can occur in a cubic enclosure. Figure 1 shows the structures that most often prevail at low Rayleigh-Darcy numbers. The first of these structures, referred to as S2, consists of a single two-dimensional roll oriented along the *z*-direction. This structure appears as the conductive state undergoes a bifurcation, or loss of stability, when the Rayleigh-Darcy number reaches $4\pi^2$. At this point, the pore-air closest to the lower heated wall expands and rises as it reaches the vertical wall. It then travels horizontally, releases heat to the upper wall, and descends along the other vertical wall. The second structure, referred to as S3, is much more complex and unstable at Rayleigh-Darcy numbers smaller than 50 (Sezai, 2005). The flow at the top surface of the cube moves away from the hot upwelling plumes and inwards into the cold plummeting flow. The hot upwelling plumes (and cold plummeting flow) are at diagonally opposite corners of the cube. As pointed out by Straus and Shubert (1979), the upwelling plumes are narrow and the plummeting flows cover a broad portion of the cross-sectional area in the lower portions of the cube. The third structure, referred to as S4, consists in two counter-rotating, two-dimensional rolls, oriented along the *z*-direction. As pointed out by Sezai (2005), this structure only prevails at Rayleigh-Darcy numbers larger than 130 and smaller than 290.

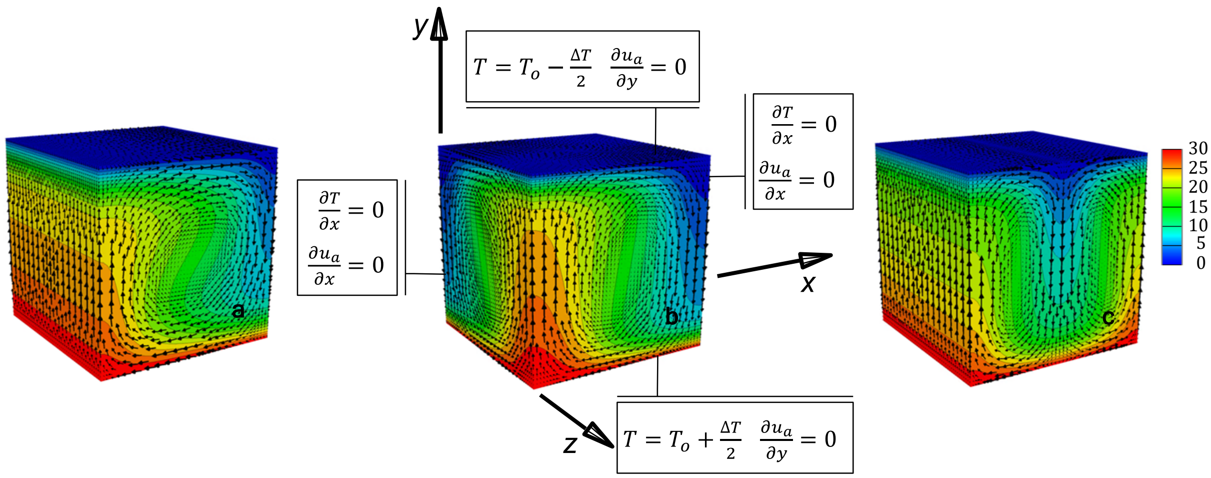

Figure 1. Temperature contour plot, vectors and boundary conditions, Ra_D = 150. (a) Structure S2, (b) Structure S3, (c) Structure S4.

Figure 2a shows the temperature profile at the center of the cubic enclosure in the presence of an S2 structure at Rayleigh-Darcy numbers of 50, 100, 150, and 180. The temperature profile is close to linear at smaller Rayleigh-Darcy numbers, when conduction is predominant, and becomes sigmoidal (or S-shaped) at larger Rayleigh-Darcy numbers, when convection prevails. At these larger Rayleigh-Darcy numbers, the interior of the cube approaches isothermal conditions and the thermal gradients are largest in the boundary layers at the top and bottom of the cube, where heat is transferred from the porous media to the boundary. Figure 2b shows the mean Nusselt number of the steady-state S2 structure at various Rayleigh-Darcy numbers. As expected, the model captures the onset of steady-state convection that occurs as the Rayleigh-Darcy number reaches $4\pi^2$. The numerical results are also shown to be in very close agreement with those obtained by Shubert and Straus (1979). The difference between the two sets of results translates into an average percent difference of approximately 1%. It should be noted that these results are in perfect agreement with those found in a square enclosure in which the vertical walls are impervious and rigid barriers to heat transfer, and thus numerically equivalent to the Horton-Rogers-Lapwood problem.

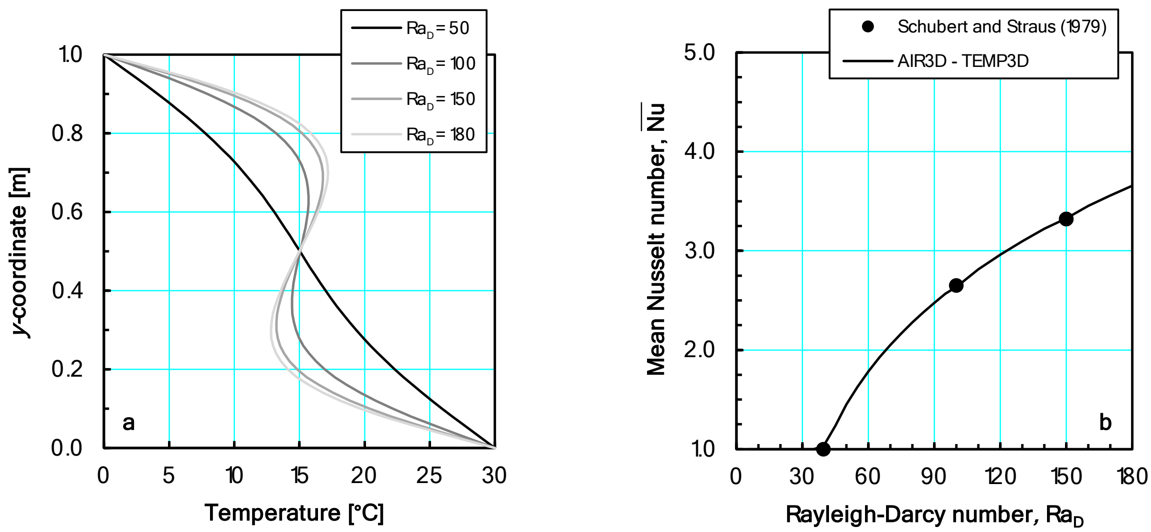

Figure 2. Results of the benchmark problem with an S2 structure. (a) Temperature profile at the center of the cube. (b) Relationship between the mean Nusselt number and the Rayleigh-Darcy number.

Summary and Conclusions

The objective of this example was to demonstrate the ability of coupled AIR3D and TEMP3D analyses to solve multi-dimensional problems involving natural convection. The capabilities of the software were assessed with a widely used benchmark for natural convection in a cube. The results were shown to capture the onset of steady-state convection that occurs as the Rayleigh-Darcy number reaches $4\pi^2$. The results were also shown to be in very close agreement with those of a reference numerical analysis.

References

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