GeoStudio Example File

Verification – Water convection in a cube

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Introduction

Convection is ubiquitous in nature and plays an important role in numerous engineering and earth science problems. It occurs as pore fluid moves within the soil. The movement of the fluid can be generated by external forces (such as pressure), or then again, by the forces of gravity acting on fluid density variations. These fluid density variations may result from non-uniform temperature fields and/or non-uniform concentration of any of the fluid components. In the presence of a non-uniform temperature field, most fluids become lighter and rise when heated, and become denser and fall when cooled. These movements inevitably alter the thermal regime, affect the fluid density, and create a coupled system. The manner in which this buoyancy-driven flow occurs depends on the boundary conditions and the geometry (or shape) of the domain.

The objective of this example is to verify the heat and mass transfer formulation against a widely used benchmark for natural convection of pure water in a porous cube. In this problem, the cubic enclosure is heated from below and cooled from the top while the vertical walls are considered impervious and rigid barriers to heat transfer. The exercise lends credibility to the use of the software for multi-dimensional problems involving natural convection.

Numerical Simulation

The purpose of the example is to model natural convection in a porous cube. The density of pure water is mostly a function of temperature. As with most fluids, pure water expands and becomes lighter when heated. The density of pure water is assumed to conform to Thiesen's formula, and all other fluid properties are assumed to remain constant. The temperature of the horizontal walls of the enclosure are equally offset with respect to a mean reference temperature. The fluid is initially hydrostatic, and the temperature is representative of a slightly disturbed conductive state. The domain is discretized in a structured mesh of 50×50×50 hexahedral elements, and exponential time stepping schemes are used in the transient analyses. These analyses are deemed to have reached steady-state conditions when the differences between the heat fluxes at the upper and lower walls is less than 2%.

The Rayleigh-Darcy dimensionless number characterizes the flow regime within the cubic enclosure (Lapwood, 1948):

$$Ra_D \equiv RaDa\frac{k_{t,w}}{k_t} = \frac{g\beta_{t,o_w} H^4 \Delta T/H \;\; \mu_{w_o}/\rho_{w_o} \;\; k\frac{\mu_{w_o}}{\rho_{w_o} g} \;\; k_{t,w}}{\left(\mu_{w_o}/\rho_{w_o}\right)^2 \;\; k_{t,w}/C_{t,w} \;\; H^2 \;\; k_t} \qquad \text{Equation 1}$$

where Ra, Da are the Rayleigh and Darcy dimensionless numbers, respectively; $k_{t,w}$ is the thermal conductivity of pure water; k_t is the thermal conductivity of the porous media; g is the gravity constant; $\beta_{t,o_w} = -\frac{1}{\rho_{w_o}}\left(\partial\rho_w/\partial T\right)_{P,C}$ is the reference volumetric thermal expansion coefficient of pure water at constant pressure and concentration; H is the reference dimension, which is taken equal to the height of the enclosure; ΔT is the temperature differential; μ_{w_o} is the reference dynamic viscosity of pure water; ρ_{w_o} is the reference density of pure water; $C_{t,w}$ is the volumetric heat capacity of pure water at constant pressure and temperature; and k is the hydraulic conductivity of the porous media. In order to obtain results at various Rayleigh-Darcy numbers, the transient analyses are set up with different values of hydraulic conductivity and constant thermal material properties, such that:

$$k = \frac{\left(\mu_{w_o}/\rho_{w_o}\right)^2 \;\; k_{t,w}/C_{t,w} \;\; H^2 \;\; k_t}{g\beta_{t,o_w} H^4 \Delta T/H \mu_{w_o}/\rho_{w_o} \frac{\mu_{w_o}}{\rho_{w_o} g} k_{t,w}} Ra_D \qquad \text{Equation 2}$$

where μ_{w_o} = 1.139×10^{-3} N s/m^2, ρ_{w_o} = 999.103 kg/m^3, g = 9.807 m/s^2, β_{t,o_w} = 1.509×10^{-4} 1/K, H = 1 m, ΔT = 1 K = 1 °C, $k_{t,w}$ = 0.600 J/s/m/°C, $C_{t,w}$ = 4.182×10^6 J/m^3/°C, k_t = 1.312 J/s/m/°C. The influence of the flow regime is measured with the mean Nusselt dimensionless number $\bar{N}u$, which is defined as the ratio of convective to conductive heat transfer normal to the upper boundary:

$$\bar{N}u = -\frac{\frac{1}{B}\left(\int_{x=0}^{B} \frac{\partial T}{\partial y} dx\right)_{y=H}}{\Delta T/H} \qquad \text{Equation 3}$$

where B is the width of the enclosure.

Results and Discussion

A number of steady-state convective flow structures (or patterns) can prevail in a cubic enclosure. Figure 1 shows the most prevalent structures that occur at low Rayleigh-Darcy numbers. The first of these structures, referred to as S2, consists in a single two-dimensional roll oriented along the *z*-direction. The S2 structure possess a bifurcation, or loss of uniqueness, at a critical Rayleigh-Darcy number of $4\pi^2$. At larger values of the Rayleigh-Darcy number, the volumetric fluxes are larger than zero, and the water rises when heated. It then travels horizontally as it releases heat to the upper surface of the cube, and descends along the right-hand side wall. The second structure, referred to as S3, is much more complex and unstable at Rayleigh-Darcy numbers smaller than 50 (Sezai, 2005). The flow at the top surface of the cube moves away from the hot upwelling plumes and inwards into the cold downward flow. The hot upwells (and cold downward flow) are at diagonally opposite corners of the cube. As pointed out by Straus and Shubert (1979), the upwells are narrow and the descending flows cover a broad portion of the cross-sectional area in the lower portions of the cube. The third structure, referred to as S4, consists in two counter-rotating, two-dimensional rolls, oriented along the *z*-direction. As pointed out by Sezai (2005), this structure only prevails at Rayleigh-Darcy numbers larger than 130 and smaller than 290.

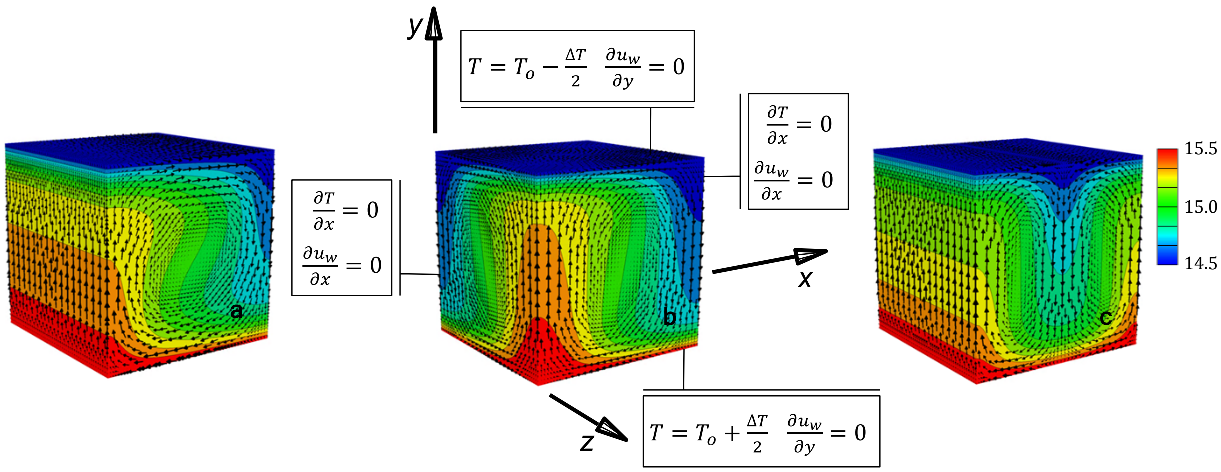

Figure 1. Temperature contour plot, vectors and boundary conditions, Ra_D = 150. (a) Structure S2, (b) Structure S3, (c) Structure S4.

Figure 2a shows the temperature profile at the center of the cubic enclosure in the presence of an S2 structure at Rayleigh-Darcy numbers of 50, 100, 150, and 180. As expected, the temperature profile is close to linear at small Rayleigh-Darcy numbers, when conduction is predominant, and evolves into an S-shaped curve as the Rayleigh-Darcy number increases and convection prevails. At these larger Rayleigh-Darcy numbers, the interior of the cube approaches isothermal conditions and the thermal gradients are largest in the boundary layers at the top and bottom of the cube, where heat is transferred from the porous media to the boundary. Figure 2b shows the Nusselt number of the steady-state S2 structure at various Rayleigh-Darcy numbers. As anticipated, the model captures the onset of steady-state convection that occurs as the Rayleigh-Darcy number reaches the critical threshold of $4\pi^2$. The numerical results are also shown to be in very close agreement with those obtained by Shubert and Straus (1979). The difference between the two sets of results translates into a maximum percent difference of approximately 1%. It should be noted that these results are in perfect agreement with those found in a square enclosure in which the vertical walls are impervious and rigid barriers to heat transfer, and thus numerically equivalent to the Horton-Rogers-Lapwood problem.

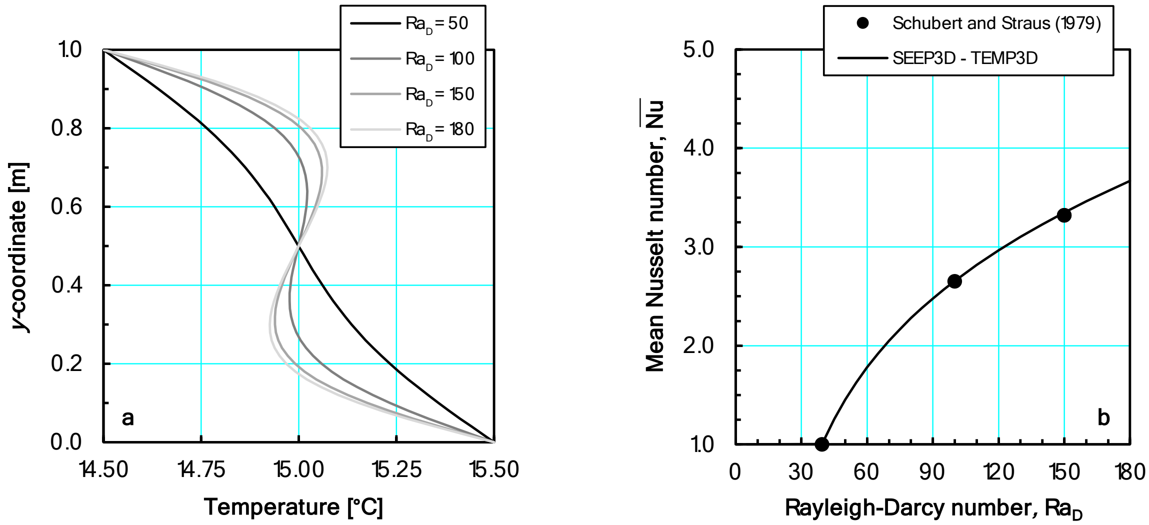

Figure 2. Results of the benchmark with an S2 structure. (a) Temperature profile at the center of the cubic enclosure. (b) Relationship between the mean Nusselt number and the Rayleigh-Darcy number.

Summary and Conclusions

The objective of this example was to demonstrate the ability of coupled SEEP3D and TEMP3D analyses to solve multi-dimensional problems involving natural convection. The capabilities of the software were assessed with a widely used benchmark for natural convection in a cubic enclosure. The results were shown to capture the onset of convective flow that occurs as the Rayleigh-Darcy number reaches $4\pi^2$. The results were also shown to be in very close agreement with those of a reference numerical analysis.

References

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