GeoStudio Example File

Convective Surface on a Semi-Infinite Domain

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Introduction

A convective surface provides a possible boundary condition for a heat transfer analysis. The convection surface boundary condition embodies energy transfer by diffusion (conduction) from a bounding surface into a moving fluid (advection). A convective surface boundary condition can be used, for example, to represent the presence of a freeze pipe in a simulation of artificial ground freezing or other processes involving the flow of fluid over or within a bounding surface.

Convective surface type boundary conditions are represented by a heat flux (q) that is calculated according to Newton's Law of cooling. The calculated heat flux is a function of the convection heat transfer coefficient, simulated ground temperature, and input fluid temperature. In the physical reality, the convection heat flux into or out of a surface varies smoothly and rapidly as the temperature at the surface changes. In a numerical analysis, the convection heat flux is an instantaneous value that is calculated based on the conditions at the beginning of the time step. A conventional approach to implementing the boundary condition assumes the heat flux constant over the time step, which generally causes over-cooling or over-heating depending on the role of the boundary condition. Numerical oscillation then initiates on the subsequent time step and the results become irrelevant.

TEMP/W uses a special procedure to accommodate the non-linearity of the convective surface boundary condition and promote numerical stability. The objective of this example is to verify the implementation of the convective surface boundary condition by comparing the simulated results to a closed-form solution.

Numerical Experiments and Closed Form Solution

The model domain comprises a 10 m column of elements having a global element size of 0.05 m and an initial temperature of 20 °C. The lower boundary is held at a constant temperature of 20 °C and is assumed to be located far enough from the bounding surface to deem the domain semi-infinite.

The medium is assumed to be characterized by a thermal conductivity and volumetric heat capacity of 0.52 W/m/K and 3,772,000 J/m^3/°C, respectively (Define | Materials). The surface of the domain is subjected to a sudden change of conditions that are produced by a fluid flowing over the surface at a constant temperature of – 15 °C. The convection heat transfer coefficient, which embodies many energy transfer processes at the surface, is assumed constant at 40 J/s/m^2/°C (Define | Boundary Conditions).

The response of the domain was simulated for a 1-day duration (86,400 seconds) using two cases: the first case used 20-time steps and the second case used 10-time steps. Both analyses use an exponential time sequence (Define | Analyses). As previously noted, the heat flux occurring at a convective surface is highly non-linear in the physical world because the temperature at the surface changes rapidly. The simulations were completed using an initial time increment size of 200 seconds and 3600 seconds for case 1 and 2, respectively, to explore the role of time stepping in capturing the non-linearity of the boundary condition.

The latent heat of fusion for water was set to 0.0 J/m3 in the project settings because the analytical solution does not accommodate phase change (Define Analyses | Heat tab).

Results and Discussion

Figure 1 and Figure 2 show the temperature at the top of the semi-infinite domain and the cumulative energy leaving the domain, respectively. The simulated response is compared to an analytical solution (Carslaw and Jaeger, 1986). A numerically stable solution was obtained regardless of the initial time step size; however, the surface temperature was slightly elevated during the early stages of the simulation for case 2, which used a larger initial increment size (Figure 1). The simulated cumulative energy leaving the column is slightly overestimated in the early part of the simulation for case 2 because the surface temperature was over-estimated, which leads to a larger difference between the fluid and ground temperatures. Regardless, both the temperature and energy time histories can be considered to accurately match the analytical solution. In addition, the numerical stability of the boundary condition implementation allowed for larger time steps to be used, which reduces the computational cost.

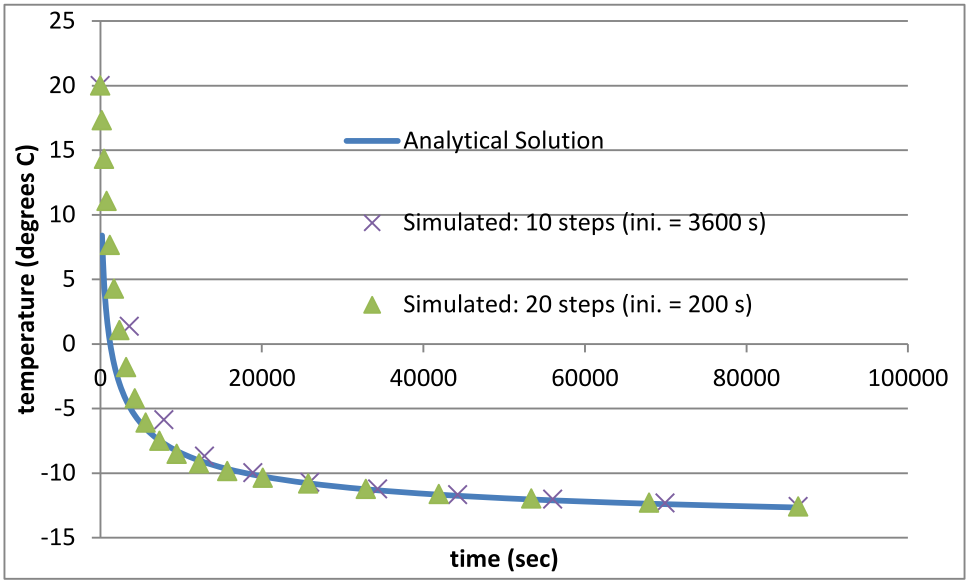

Figure 1 Temperature versus time at the top of the semi-infinite domain: analytical solution versus simulated response.

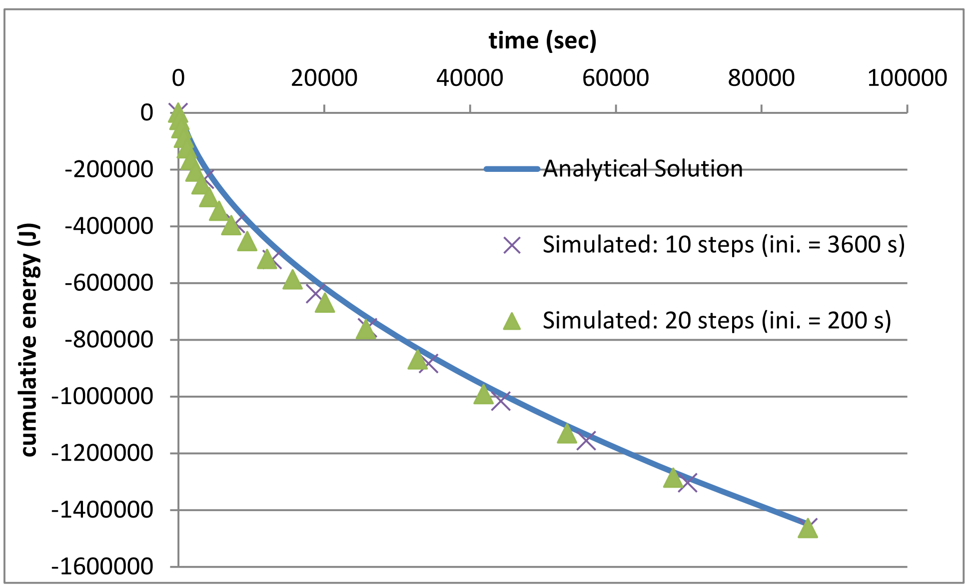

Figure 2 Cumulative energy leaving the top of the semi-infinite domain: analytical solution versus simulated response.

Summary and Conclusions

In this example, TEMP/W is benchmarked against a closed-form analytical solution for a problem involving a semi-infinite domain subject to a convective surface boundary condition.

The simulated response matches closely the analytical solution even when larger time steps are used. The closeness of the match verifies that the boundary condition implementation is able to reasonably approximate the non-linear response of a convective surface while being numerically stable.