# Two Methods of Gradient-Enhanced Gridding – A Discussion

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**Summary**

The lateral gradient of the total field (TF or TMI) as measured by two magnetometers mounted at the wingtips of a fixed-wing aircraft, can provide more information in a total field grid than can be obtained using a single magnetometer. Gridding is essentially a process in which data taken along the flight lines is interpolated *across* the lines. Since in most interpolation methods, the across-line gradient at the flight line must be calculated, it stands to reason that using the *measured* (lateral) gradient, will lead to a more accurate image of features between the lines. How best to use the gradient is the subject of this paper. The two methods discussed are a) The Nelson Method (Nelson, 1994) and b) Pseudo-Lines (Hardwick, 1999).

The Nelson Method is based on generating the total field from the lateral and longitudinal gradients using a Hilbert transform relationship. It offers the possibility of forming a TF grid without the necessity of tie lines, since the gradients discriminate against the long-wavelength diurnal components in the TF that require tie-line levelling. However, the Hilbert transform formulation is essentially an integration of the gradients in the frequency domain and the result must be stabilized by adding at least the zero-wave number (or "DC" value) of the TF. How much more than just the DC value needs to be added, is a subject for discussion; if long-wavelength components of the TF are added, how do these components get levelled? Furthermore, as will be shown, at least one and preferably more, tie lines are the best way to evaluate (and/or regulate) the effectiveness of gradient-enhanced gridding.

The Pseudo-Line method uses the lateral gradient more directly and simply. Here, the starting point is a well-levelled total field, implying conventionally spaced tie lines. For each flight line, a pair of straddling "pseudo-lines" is generated, separated from their parent line by a small distance, ΔY. The value of the TF on the lines is extrapolated to the pseudo-lines using the measured lateral gradient. Finally, the pseudo-lines are used to form a new TF grid.

In conventional gridding, depending on the line spacing chosen, small, closed features can tend to gather on the flight lines. With both methods, some of these features can be seen to move away from the lines and more detail shows up between the flight lines. This makes both methods attractive, but care must be taken because some of these added features could be artefacts. With pseudo-lines, for example, the ΔY spacing is critical; too small a spacing will produce about the same result as normal gridding, whereas too great a spacing will produce overshoots or even ringing instabilities. The best control of these effects is to sample any new grid along the tie lines and to compare the profile with the TF profile that was actually measured along these tie lines.

This paper shows results using both methods and makes comparisons. Quality control based on tie line profiles is illustrated, as is de-skewing of the gradient measurements, which is necessary for both methods.

**The Nelson Method**

This gridding method is based on relationships set out by Misac Nabigian (Nabighian, 1984) and applied by Nelson. The relationship between total field and the horizontal components of its gradient can be expressed by a Hilbert transform in either the space or in the wave-number domain. Intuition suggests that a total field grid constructed from its gradient components should be free from diurnal errors and thus, the need for levelling by tie lines might be reduced, with the attendant reduction of flying costs.

Construction of the total field grid from the horizontal gradients is straightforward in the frequency domain:

$$FFT\left(TF\right) = \left(-i/k^2\right)\left\{k_x\, FFT\left(Gx\right) + k_y\, FFT\left(Gy\right)\right\} \qquad (1)$$

where

$k_x$ and $k_y$ are the wave numbers in the respective x and y directions,

$k = \sqrt{k_x^{\,2} + k_y^{\,2}}$ ,

$i = \sqrt{-1}$ and

TF, Gx and Gy are respectively the total field, longitudinal and lateral gradient grids [1]

However, a gradiometer is by its nature a high pass filter and as such, does not contain the long-wavelength features of the total field. If a high-passed representation of the total field is all that is required (somewhat analogous to a first derivative grid), where short-wavelength features such as kimberlites are of primary interest, this is not a problem. If a full-wavelength grid of the total field is required, the long-wavelength components have to be added, but without tie line levelling, long-wavelength components of diurnal variation can distort the result. Various schemes can be used to alleviate this levelling problem, such as the use of cross-line decorrugation filters and using corrections from a well-positioned ground magnetometer.

This technique was tested on data from a survey of the Bay of Fundy, done by the NRC Convair 580 aircraft (Hardwick, 1997). To obtain reasonable long-wavelength features, (without using tie lines), we found that a low-pass bandwidth of the TF grid at 40 km was necessary. This data was added in the complementary filter arrangement shown in Figure 1. Because the survey was flown with conventionally spaced tie lines, it was possible to compare a non-tie line Nelson result with conventional gridding. Figure 2a compares the grids. Figure 2b is a way to assess

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[1] The axis convention used here is that used in aerodynamics, namely X forward, Y lateral to the right and Z down, to form a right-hand system.

the fidelity of the Hilbert process, where the high-passed leg of the complementary filter is compared to a high-pass of the normal TF grid at the filter break wavelength. It is significant to note that during the two southern-most flights, there was high diurnal and micropulsation activity, which is evident in the normal grid, but not in the gradient synthesis, demonstrating one advantage of the method. Ignoring the noisy part of the normal grid, it can be seen that the medium-wavelength features match pretty well, while there might be some question about the shortest wavelengths in places. (The differences in long-wavelength colouring are corrected by the final low-pass additions).

This test evaluated the effectiveness of "levelling" the low-passed total field *without* the use of tie lines. Figure 2c shows the 40-km low-passed total field from normal tie line levelling and the same low-pass corrected only base station subtraction and by an 8-km cross-line decorrugating filter. There is reasonable agreement, showing that the necessary long-wavelength addition can be "levelled" without tie lines.

**Pseudo-Line Gridding**

Straightforward cross-line gridding using interpolation methods such as Akima or cubic spline, use the cross-line gradient as estimated by differencing the TF values on adjacent lines. It stands to reason that using the measured lateral gradient on the line should be better than using the line-to-line estimate, but early attempts to use it directly in the interpolation produced wildly varying results. Pseudo-line gridding gave a method of using the gradient in a controlled fashion. Pseudo-lines are positioned by

$$TFps = TFline + Gy * (\pm)\Delta Y \qquad (2)$$

where TFps is total field value interpolated to the pseudo-line,
TFline is the on-line value of total field
Gy is measured lateral gradient in nT/m
$\Delta Y$ is the distance to the pseudo-line

One has to ask, how accurate are the estimated pseudo-line total field values? Obviously, the linear extrapolation of the total field using the lateral gradient can only be accurate for a limited distance from the lines. In practice, this question is settled quite simply by interpolating across the survey lines using different offset distances and comparing the result with the true cross-line total field as measured along a tie line. It has to be a tie line with high, short-wavelength activity, because the procedure is only effective in a spectrum of short wavelengths and certainly not at long wavelengths. The pseudo-line offset distance is adjusted until the interpolated result most closely matches the total field along the tie line. Too small an offset results in the anomalies being under-sampled, as in conventional interpolation, while too large an offset results in overshoots.

The first truly operational use of pseudo-line gridding was for a survey flown with the NRC Convair in 1998 north of Greenland and Ellesmere Island, part of a multi-year project to map the continental margins of the Arctic Ocean (Forsyth et al., 1998). The line spacing was three km, flown at 300 m above sea level. The small-area sample in Figure 3 is typical of the improvement achieved throughout the whole survey in moderately active areas. It can be seen that the gradient-enhanced grid provides much more continuity of the larger anomalies, while the medium- sized ones appear to coalesce into more meaningful structures. Even the very small isolated anomalies that do not unite are shown in slightly more detail with gradient-enhancement. Data from the survey as far back as 1991 were processed in this way. Interpreters stated that the enhanced data revealed important dike structures that matched published data from on-shore Greenland.

Figure 4 shows a tie line from the survey that was used to set the pseudo-line $\Delta Y$ spacing. From the sharp, negative-going feature, it can be seen that the black trace for 400 m offset comes very close to matching the measured total field (red). The blue 150-m offset trace tends towards matching for all long wavelength anomalies, and certainly does a better job than the normally gridded profiles (green). Note that the 400 m offset is just starting to overshoot on the small, sharp, positive-going anomalies. The 400-m offset was chosen as a good compromise between tracking fidelity and overshoot.

Experiments with pseudo-line gridding on other projects by various agencies have shown that for an area that is adequately sampled in terms of line spacing vs. average target depth, the same resolution can be obtained with about a 33% increase in line spacing with pseudo-line enhancement (Marcotte et al., 1990; Hardwick, 1996), but to date, there are no reports of wider line spacing having been attempted in production surveys.

**Limits of Improvement with Gradient Gridding**

The improvements with gradient gridding can be seen mainly between the lines. Isolated features with wavelengths longer than line spacing show little improvement in an area that is adequately or over-sampled. The Nyquist wavelength for across-line gridding is twice the line spacing, so thinking of a "feature" as being defined by half its wavelength, we can expect cross-line features about the size of the line spacing to be resolvable with normal gridding. With gradient enhancement, somewhat smaller features can be resolved accurately. Thus, for well- or over-sampled surveys, the improvements with gradient enhancement will be seen mainly at these short wavelengths. For under-sampled surveys, improvements at somewhat longer wavelengths may be seen.

Figure 5a shows part of a well-sampled survey with a between-line feature that is better resolved by pseudo-lines. In Figure 5b the small, blue features are just at the lower limit for improvement, but with pseudo-line enhancement, they are less "bulls-eyed" on the lines and there is a bit of further development.

## Gradient Pre-Processing Requirements

It goes without saying that usable gradiometer data requires very precise compensation, much better than for single-magnetometer total field measurement. Other requirements are as follow:

Longitudinal Gradient, Gx: This quantity is needed for both gridding methods. It can be measured by a gradiometer array of a tail magnetometer and two wingtip magnetometers, but it is much easier and more accurate to derive it from

$$\mathbf{G_x} = \frac{\mathbf{dTF/dt}}{\mathbf{dX/dt}} \qquad (3)$$

where dX/dt is ground speed

This Gx is free from residual offsets and heading errors that can be present even after good compensation. It has the further advantages that it is attitude-independent and it is always referenced to the track axis of the aircraft, regardless of pitch, roll or drift angles.

Lateral Gradient Levelling: The lateral gradient is prone to offsets and long-wavelength artefacts, even with the best of compensation, and levelling is a must. Figure 6 shows the raw lateral gradient as well as the longitudinal as calculated above. The best method of removing line-to-line bias errors is to use the computed longitudinal gradient along the tie lines as a reference for levelling the lateral gradient on the flight lines. This is analogous to formal total field levelling. There will be miss-ties caused by long-wavelength artefacts that in turn, are caused usually by small drifts in the magnetometers. These are best removed by using the lateral gradient *computed* from the total field grid (where the long wavelengths will be correct) as the low-pass leg of a complementary filter, with the *measured* lateral gradient as the high-pass leg. The break wavelength should be about four or five times the line spacing. Figure 7 shows the results of these steps.

Other methods of Gy levelling are: For bias removal, fitting a polynomial to each profile and removing the zero-order component; For long-wavelength artefacts, using the TF-computed Gy below an arbitrary noise threshold and the measured Gy above, in a blending algorithm. The choice of this threshold requires a certain amount of judgement.

Geo-coding the Measured Gradients: Before gridding or any other computation, the gradients must be aligned with the geographic directions of the survey. Since the measured horizontal gradients have the unfortunate habit of changing sign with aircraft direction, their signs must be altered accordingly. This should be a simple matter, but it has been a major source of confusion in gradient processing. If a consistent convention has been established and adhered to, there should be no problem. For example: For N-S lines, *North Gradients Positive* and *East Gradients Positive* (thus, with the vertical gradient Positive Down, this forms a right-hand coordinate system). For E-W lines, *East Gradients Positive* and *SOUTH gradients Positive,* again a right-hand system. The convention must be thought out right at the start of the processing chain, where one wingtip magnetometer is subtracted from the other. One has to be particularly observant in going from one processing system to another.

Lateral Gradient Gy De-Skewing: The lateral gradient is not affected by aircraft pitch attitude and very little by roll attitude, because in normal survey flying, the average roll angle is very close to zero and minor course corrections are made at small roll angles that have minimal effect on the measured Gy. However, the drift angle (sometimes called the “crab angle”) of the aircraft can be of significance, particularly when lines are flown in a strong crosswind. The error is proportional to the size of the horizontal gradients as well as the drift angle. With a $5^o$ drift, the error will be 9% of the longitudinal gradient. Since the lateral gradient is measured in the aircraft heading axis, while Gx (as calculated above) is measured in the track axis, Gy must be corrected to the track axis by

$$Gy_{TRACK} = GX_{TRACK} \tan\beta + Gy_{HEADING} / \cos\beta \qquad (4)$$

where $\beta$ is the drift angle (Heading + Drift = Track).

The drift angle can be measured most easily by recording the magnetic compass heading and using it in the relationship above. A compass accurate to at least 0.5 degrees is necessary; this can be achieved easily and is good airmanship, quite apart from complying with aircraft operation regulations. Of course, if an aircraft has an inertial attitude heading reference system (AHARS), this would be a better source, but such systems are seldom used on a survey aircraft and a synchro-to-digital converter to read the compass is very much less expensive. Figure 8 shows the feature development with pseudo-line gridding and then effect of drift angle correction. Note the small, red, circular feature just below the + marker; with drift correction, it moves up almost a line width. This “volatility” may seem exaggerated, but for the survey lines in question, the drift angle was large, changing from $+5^o$ to $-5^o$ line to line. This illustrates the importance of drift correction.

## A Comparison of the Two Gridding Methods

In comparing the two methods of gridding, one can spend a great deal of time poring over small areas of grids. One can see many areas where gradient gridding has produced improved resolution of small anomalies. I show examples from two surveys that I consider representative. In Figure 9, a survey with N-S lines, the enhancement of a number of features can be seen with the pseudo-line gridding. Much of the same development can be seen with the Nelson grid, but the impression is that there is somewhat less of the short wavelengths. This impression continues through the rest of the examples. In the Nelson, note the two small, vertically aligned blue features that do not appear in either of the other two grids. In the middle panel, the development of features comprising a sub-parallel dike can be seen in both methods, but with slight differences and again, in the Nelson, note the blue anomaly at the end of the tail that is not seen in the other two grids. The bottom panel shows a typical small-area zoom and here, the difference in short wavelengths between the two methods is quite evident.

In the final example, Figure 10, a survey with E-W lines, it is difficult to see the difference between the two methods without careful inspection, but the same shortage of short-wavelengths is nevertheless there.

**Discussion**

An interesting experiment was to resample a Nelson grid and to perform pseudo-line enhancement on it. The result was a grid that was much closer to the "normal" pseudo-line grid. However, it would seem that in most cases, if tie lines are to be flown, it is more rational the use the pseudo-line method. Nevertheless, there may well be a scenario for the Nelson method such as this:

1. Fly a minimum of two tie lines, one in minimally low gradient for bias levelling and one in maximum gradient to validate the process.
2. Calculate Gx as above.
3. Level Gy using Gx on the low-gradient tie line to remove bias errors.
4. Form a "raw" Nelson grid (Hilbert output).
5. Make a TMI grid, low-passed at the appropriate complementary filter break wavelength.
6. Level this LP TMI grid using the tie line and possibly decorrugation.
7. Form a "final" Nelson grid.
8. Resample this grid and apply pseudo-line gridding.

A possible problem would be that Gy would not have had its long-wavelength errors removed, but if the LP TMI were well enough levelled, it could produce a *calculated* Gy grid of sufficiently sound long wavelengths that could correct the measured Gy by the complementary filter procedure described previously. With some iteration of these steps, a viable total field grid could be produced *with a minimum of tie lines*, as validatable by the high-gradient tie line.

**Conclusions**

Both the Nelson and the pseudo-line gradient gridding methods offer improved resolution of the short-wavelength features in total field grids. The pseudo-line enhancement appears to give somewhat more short-wavelength detail and if a full set of tie lines is flown, pseudo-lines would be the preferred method. However, in a scenario with reduced tie lines, as suggested above, the Nelson method, in combination with pseudo-lines, could produce a high quality total field.

Regardless of which method is used, the gradient pre-processing steps described here must be carried out with careful attention.

Both procedures are now included in some commercial software, which makes them easily accessible, so that with the increasing number of wingtip gradiometer systems being flown, data managers can be expected to take advantage of gradient enhancement.

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The "E-W" data were provided by Sander Geophysics.

Considerable processing was done by Stephen Reford of Patterson Grant and Watson.

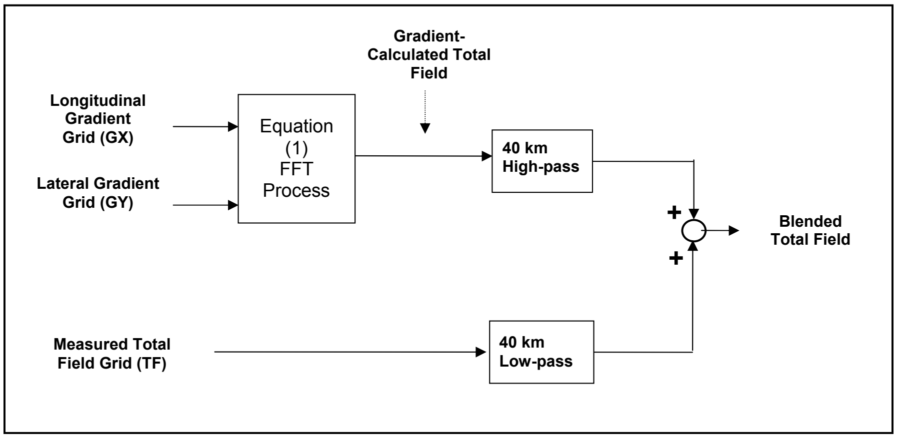
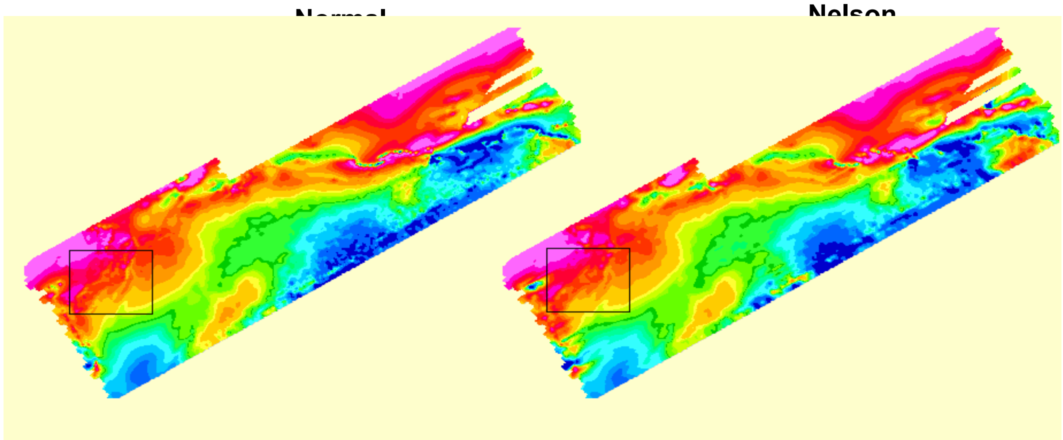



**Figure 1. Complementary Filter Blending**

Normal Nelson



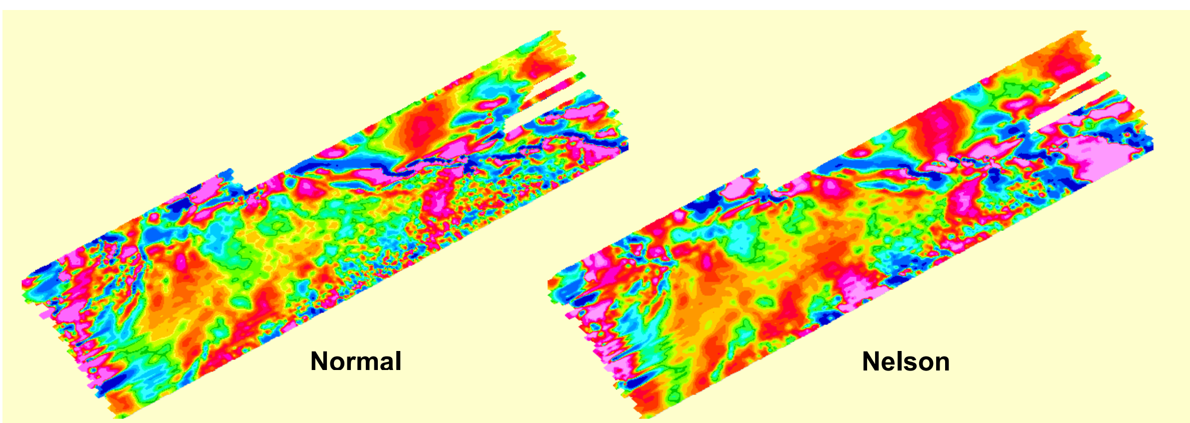



**Figure 2b. Both Grids High-Passed**

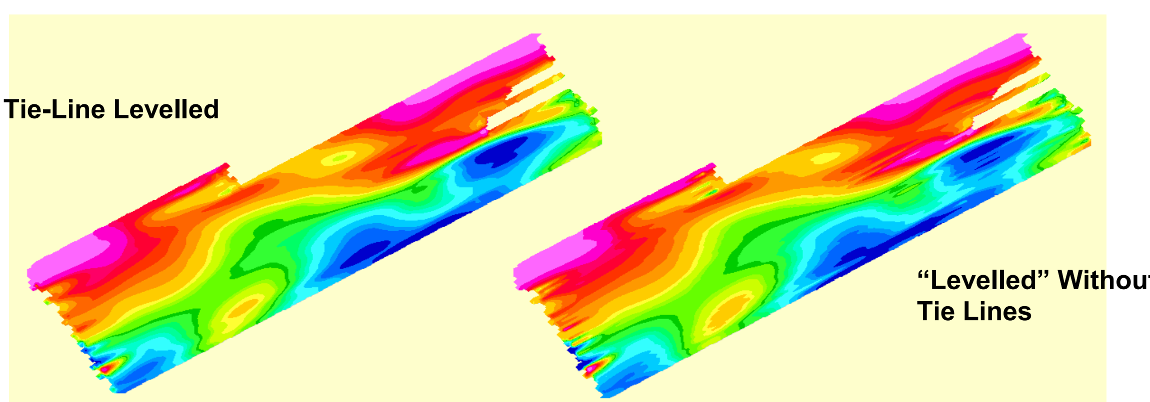



**Figure 2c. Low-passed TF Levelled**

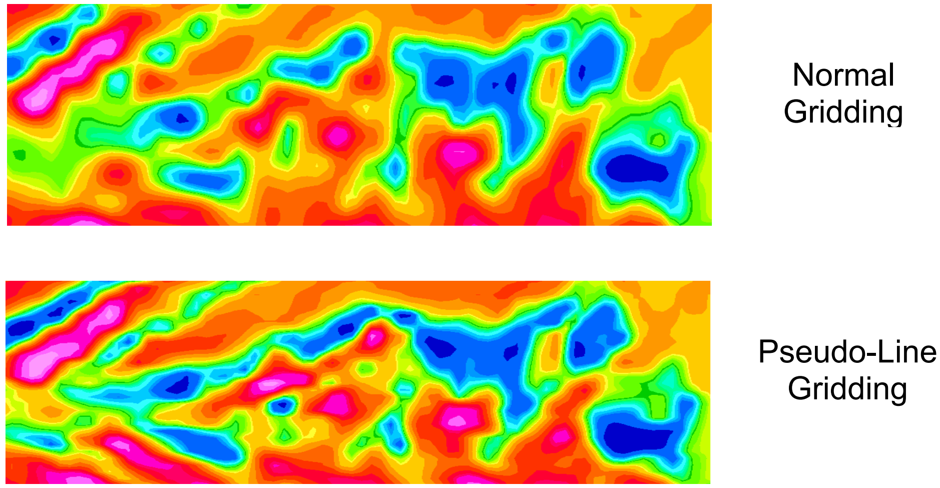



**Figure 3. Pseudo-Line Gridding (PMAP)**

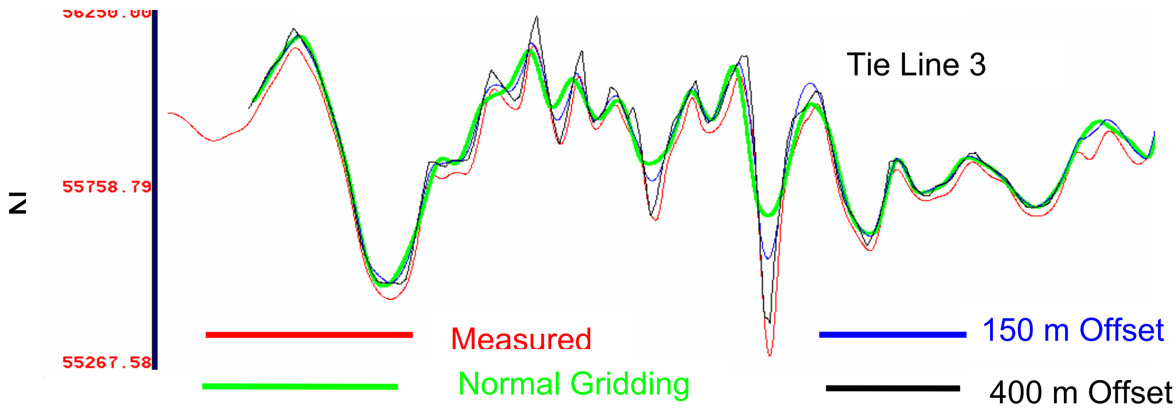



**Figure 4. Different Pseudo-Line Offsets**

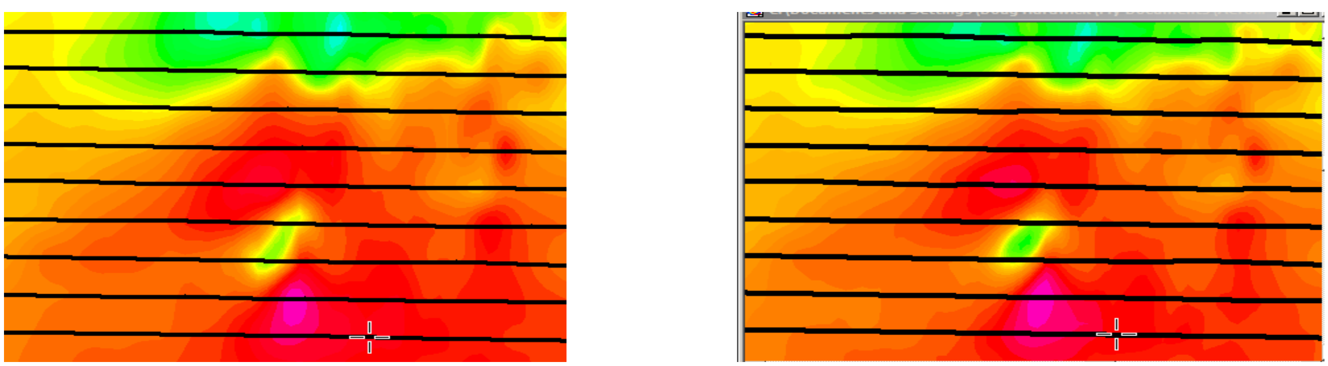


**Figure 5a. Improved Definition of a Between-Line Feature**

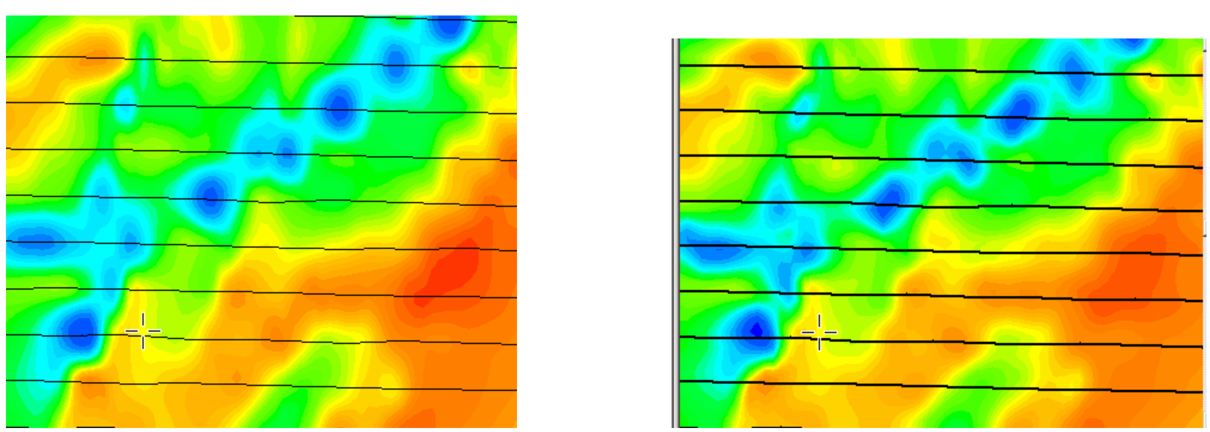


**Figure 5b. Less "Bulls Eying" with P-L Gridding**

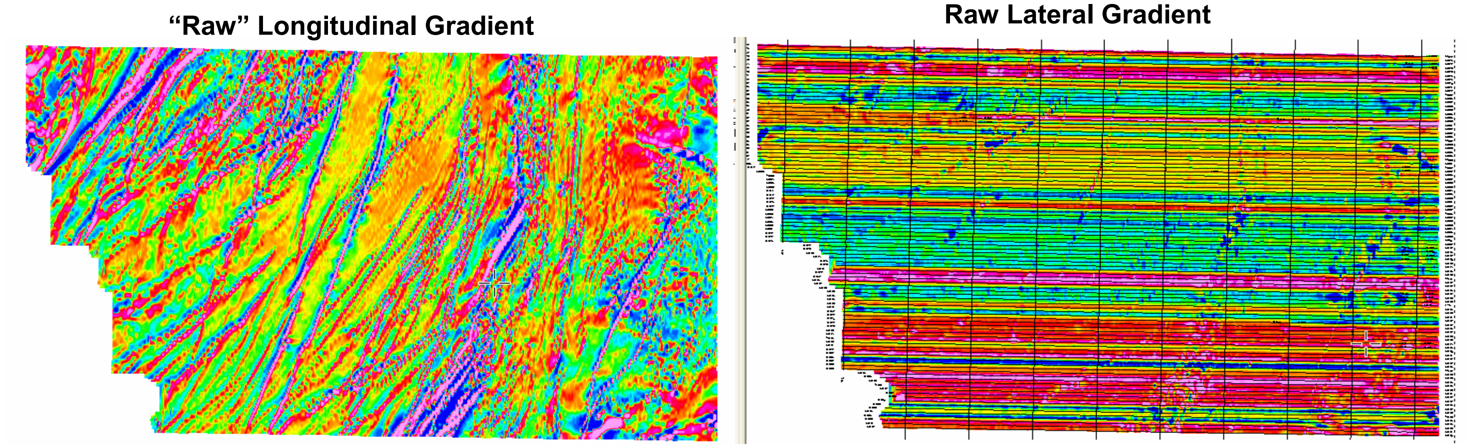



**Figure 6. Raw Horizontal Gradients**

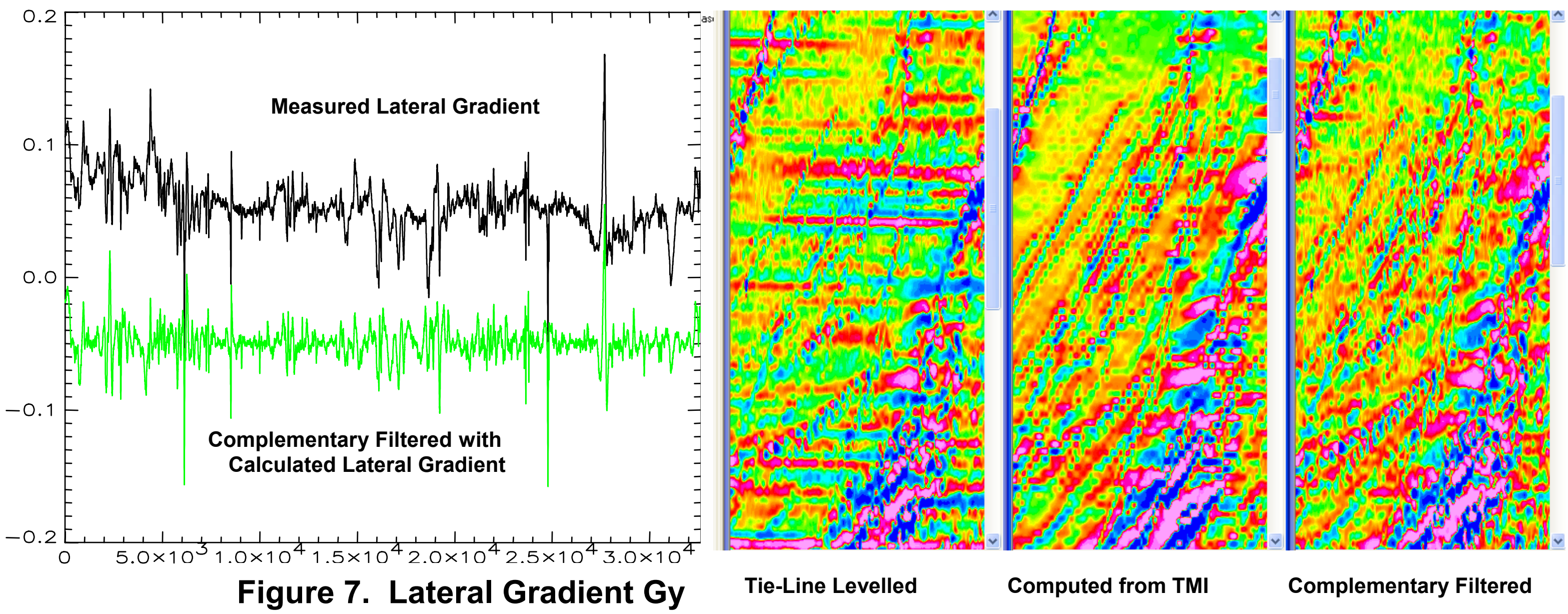



**Figure 7. Lateral Gradient Gy**

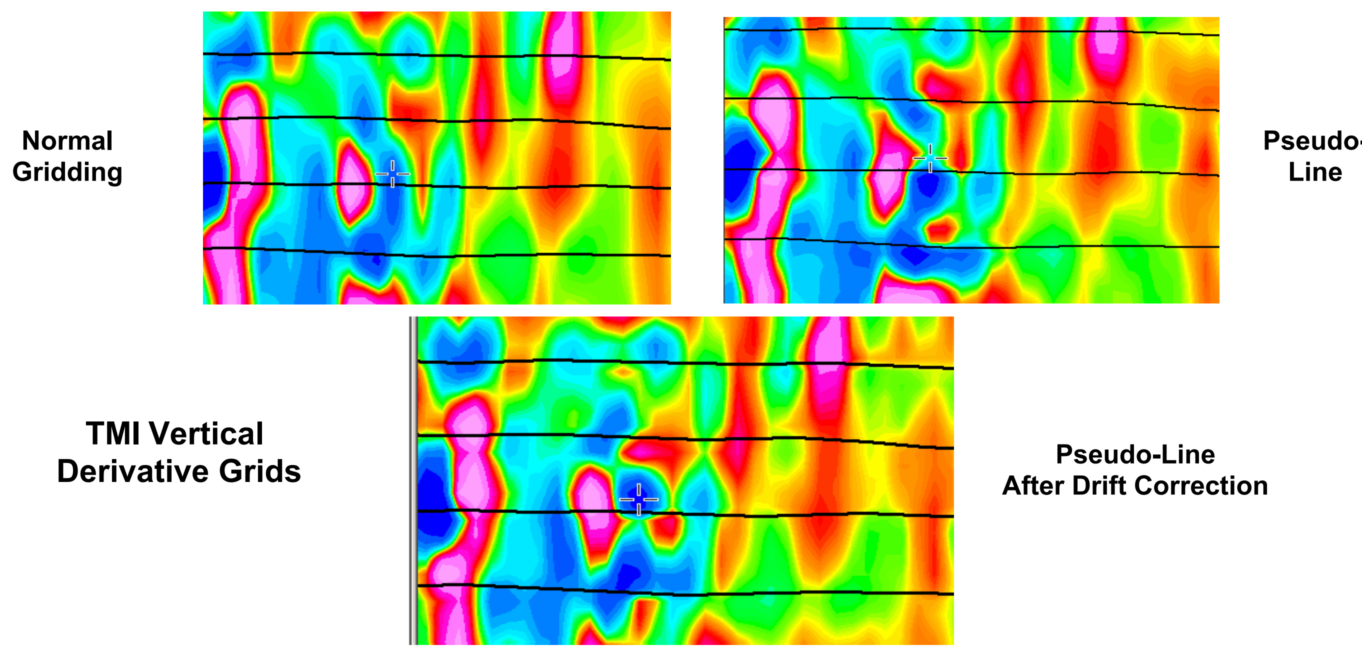
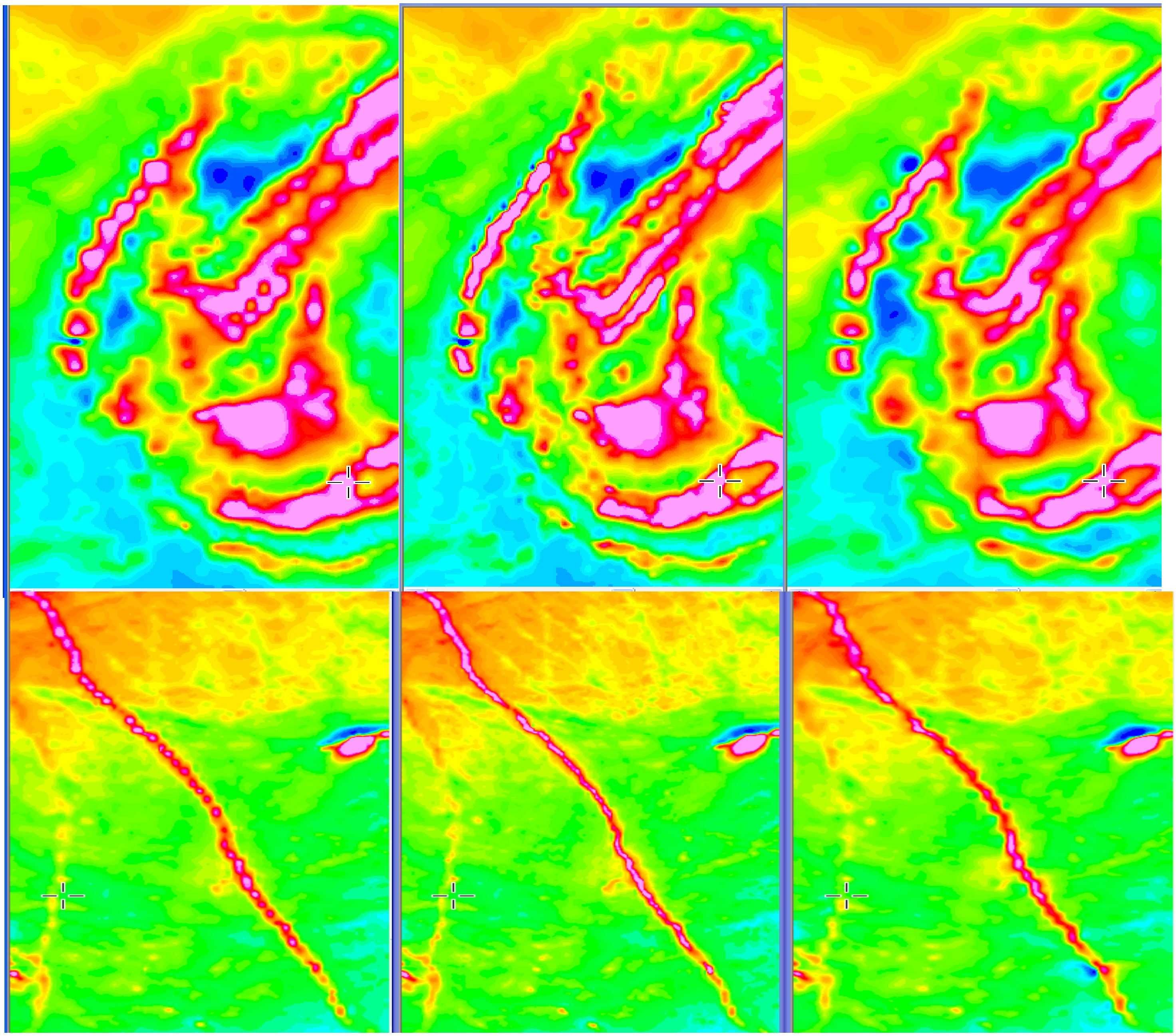



**Figure 8. Lateral Gradient Drift Correction**



**Normal** **Pseudo-Line** **Nelson**

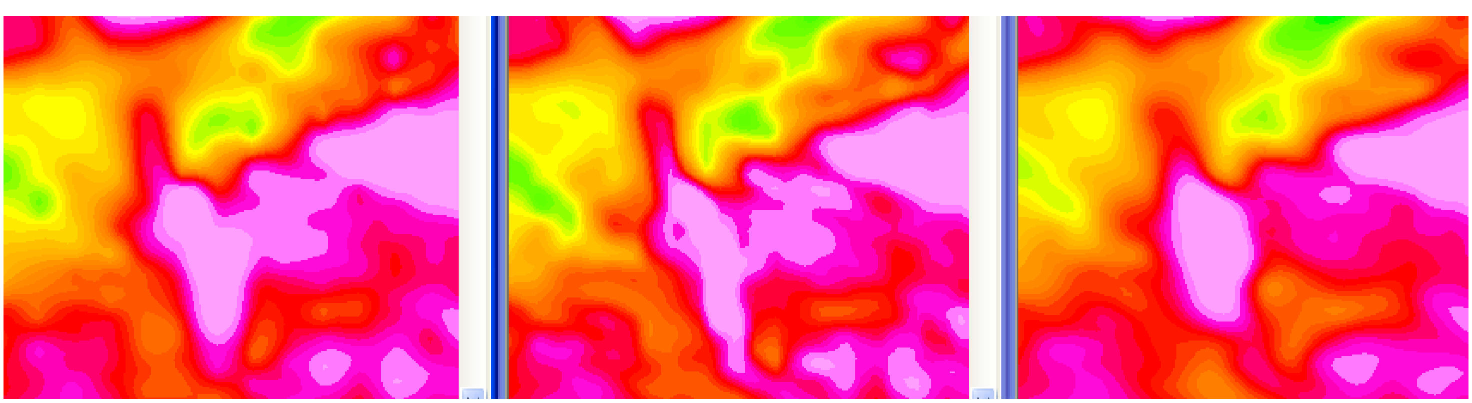
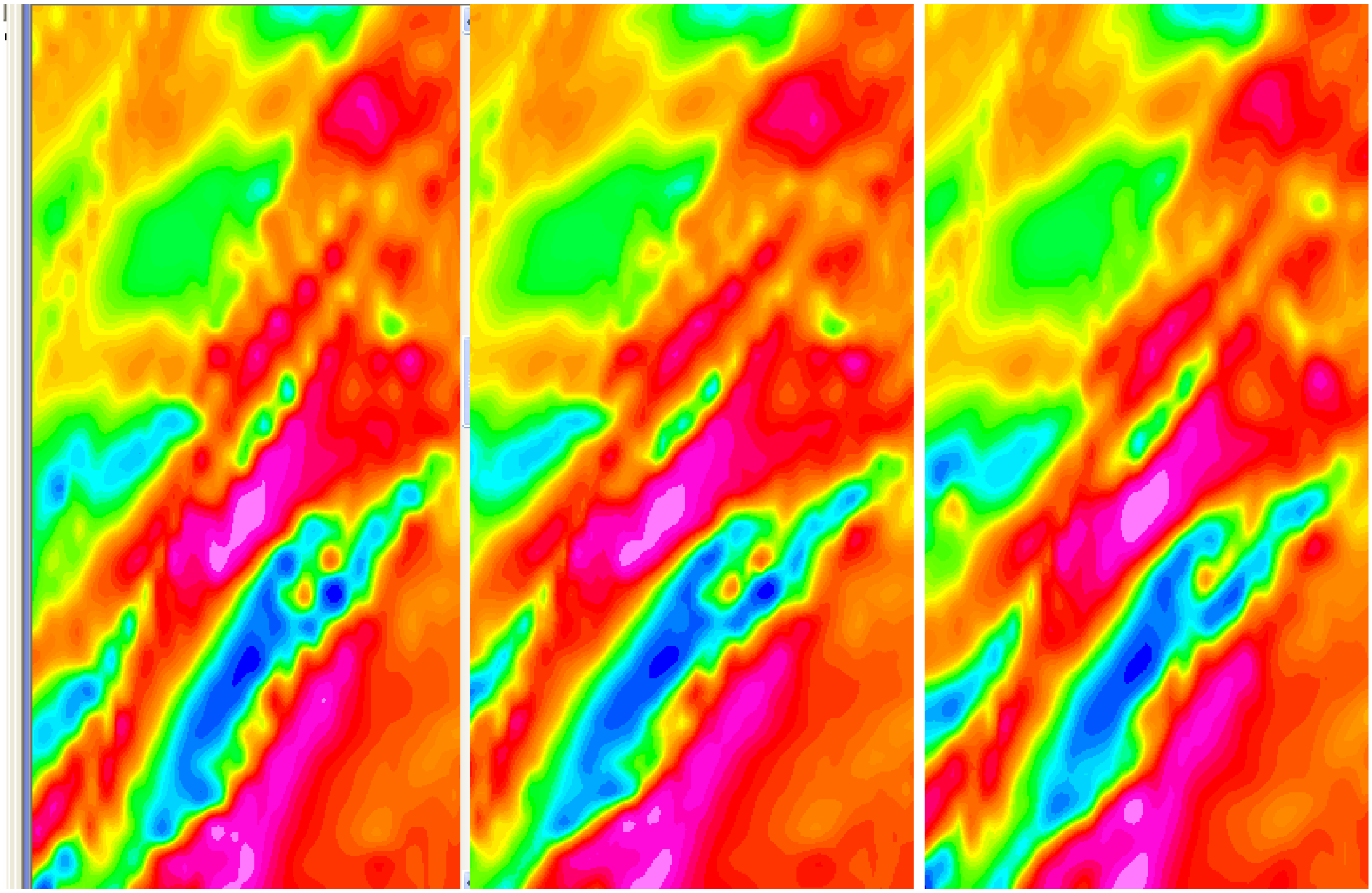


**Figure 9. Comparing Gridding Methods**



**Normal** **Pseudo-Line** **Nelson**

## Figure 10. Comparing Gridding Methods